stars of all end up as black holes, representing the ultimate in density. We will examine in turn the evolutionary roads that lead to white dwarfs, neutron stars, and black holes.

18.1 ▪ WHITE DWARFS

A white dwarf is a stellar remnant supported by electron degeneracy pressure. The name "white dwarf," as it turns out, is something of a misnomer. Nobody objects to the "dwarf" part; white dwarfs really are small compared to main sequence stars of comparable mass. As we computed in Section 13.5, the radius of the white dwarf Sirius B is $R_{wd} = 0.0084 R_\odot$, and its mass is $M_{wd} = 0.96 M_\odot$. However, not all white dwarfs are white-hot. Although the first discovered white dwarfs, such as Sirius B, have high surface temperatures (Sirius B has $T \approx 25{,}000$ K), some white dwarfs have surface temperatures as low as $T \approx 4000$ K.

White dwarfs are high in density compared to main sequence stars. The average density of Sirius B is

$$\rho_{wd} = \frac{M_{wd}}{M_\odot}\left(\frac{R_\odot}{R_{wd}}\right)^3 \rho_\odot = 0.96(0.0084)^{-3}\rho_\odot \tag{18.1}$$

$$\approx 2\times 10^6 \rho_\odot \approx 2\times 10^9\ \text{kg m}^{-3}, \tag{18.2}$$

about 200,000 times the density of lead.[2] Supporting such a dense object in hydrostatic equilibrium requires a high internal pressure. In Section 15.1, we computed the approximate central pressure for a sphere in hydrostatic equilibrium:

$$P_c \sim \frac{2GM\langle\rho\rangle}{R} \sim \frac{3GM^2}{2\pi R^4}. \tag{18.3}$$

The central pressure in a white dwarf must then be

$$P_c \sim \left(\frac{M_{wd}}{M_\odot}\right)^2 \left(\frac{R_\odot}{R_{wd}}\right)^4 P_{c,\odot} \sim (0.96)^2(0.0084)^{-4} P_{c,\odot} \tag{18.4}$$

$$\sim 2\times 10^8 P_{c,\odot} \sim 10^{18}\ \text{atm}. \tag{18.5}$$

If this pressure were provided by ordinary thermal motions, the temperature at the center of Sirius B would have to be $T_c \sim 6\times 10^9$ K. However, the pressure inside a white dwarf is *not* due primarily to thermal motions. Instead, it is provided by the degenerate electrons.

18.1.1 Degeneracy Pressure

As noted in Section 17.2, electrons become **degenerate** when they are packed closely enough that the Pauli exclusion principle produces an additional form of pressure to keep

[2] This might seem more impressive if we point out that 2×10^9 kg m^{-3} is equivalent to 10 tons per teaspoon.

them apart.[3] The **electron degeneracy pressure** is a consequence of the Heiser uncertainty principle, which states that you can't simultaneously specify the position x and momentum p of a particle to arbitrary accuracy. There is always an uncertainty in each such that

$$\Delta x \Delta p \geq \hbar, \tag{18.6}$$

where $\hbar$ is the reduced Planck constant introduced in Section 5.1. Suppose that the degenerate electrons have a number density n_e. In their cramped conditions, each electron is confined to a volume $V \sim n_e^{-1}$. Thus, the location of each electron is determined with an uncertainty $\Delta x \sim V^{1/3} \sim n_e^{-1/3}$. From the uncertainty principle, the minimum uncertainty in the electron momentum is

$$\Delta p \sim \frac{\hbar}{\Delta x} \sim \hbar n_e^{1/3}. \tag{18.7}$$

If the electrons are nonrelativistic,

$$\Delta v = \frac{\Delta p}{m_e} \sim \frac{\hbar n_e^{1/3}}{m_e}, \tag{18.8}$$

where m_e is the mass of the electron.

Thanks to the uncertainty principle, degenerate electrons are zipping around with a speed $v_e \propto n_e^{1/3}$ regardless of how low the temperature drops. These "Heisenberg speeds" contribute to the pressure, just as the thermal speeds do. For ordinary thermal motions, the electron speeds are

$$v_{th} \sim \left(\frac{kT}{m_e}\right)^{1/2}, \tag{18.9}$$

and the pressure contributed by thermal motions of electrons is

$$P_{th} = n_e kT \sim n_e m_e v_{th}^2. \tag{18.10}$$

By analogy, the "Heisenberg speeds" contribute a pressure

$$P_{degen} \sim n_e m_e (\Delta v)^2 \sim n_e m_e \left(\frac{\hbar n_e^{1/3}}{m_e}\right)^2 \sim \hbar^2 \frac{n_e^{5/3}}{m_e}. \tag{18.11}$$

We label a population of electrons as "degenerate" when $P_{degen} > P_{th}$.

[3] Electrons, neutrons, and protons are all fermions, particles with half-integral spin, to which the Pauli exclusion principle applies. Photons are examples of bosons, particles with integral spin, to which the exclusion principle does not apply.

18.1.2 Mass–Radius Relationship

Because white dwarfs are supported by electron degeneracy pressure, we can derive a simple mass–radius relation for white dwarfs. (Warning for the faint-hearted: since we want only the correct proportionality between radius and mass, and are not concerned with exact numbers, we'll be omitting numerical factors like π and 2 from the following analysis.) Since a white dwarf is in hydrostatic equilibrium (equation 18.3), its central pressure must be

$$P_c \sim \frac{GM^2}{R^4}. \qquad (18.12)$$

This is true for *all* spheres in hydrostatic equilibrium, regardless of the pressure source. If the pressure is provided by degenerate electrons, then from equation (18.11),

$$P_c \sim \hbar^2 \frac{n_e^{5/3}}{m_e} \sim \hbar^2 \frac{\rho^{5/3}}{m_p^{5/3} m_e} \sim \frac{\hbar^2}{m_p^{5/3} m_e} \frac{M^{5/3}}{R^5}. \qquad (18.13)$$

(Since a carbon/oxygen white dwarf is made of ionized "metals," its mean molecular mass is $\mu \approx 2$. In keeping with our policy of ignoring small numerical factors, we have set $2 \approx 1$.) Equating the pressure required for hydrostatic equilibrium (equation 18.12) with the pressure provided by degenerate electrons (equation 18.13), we find that

$$G\frac{M^2}{R^4} \sim \frac{\hbar^2}{m_p^{5/3} m_e} \frac{M^{5/3}}{R^5}, \qquad (18.14)$$

or

$$R \sim \frac{\hbar^2}{G m_e m_p^2} \left(\frac{M}{m_p}\right)^{-1/3}. \qquad (18.15)$$

Notice the counterintuitive result that more massive white dwarfs have a smaller radius. We don't expect 40 pounds of cow manure to fit in a smaller bag than 20 pounds of cow manure, but we do expect a $1M_\odot$ white dwarf to fit in a smaller volume than a $0.5M_\odot$ white dwarf. Since larger masses correspond to smaller radii, we expect the average density to increase rapidly with mass:

$$\langle\rho\rangle \sim \frac{M}{R^3} \sim \frac{G^3 m_e^3 m_p^5}{\hbar^6} M^2. \qquad (18.16)$$

The exact value of the radius for a white dwarf of a given mass depends slightly on the chemical composition of the dwarf. However, a good empirical fit is found to be

$$R \approx 0.01 R_\odot \left(\frac{M}{0.7M_\odot}\right)^{-1/3}. \qquad (18.17)$$

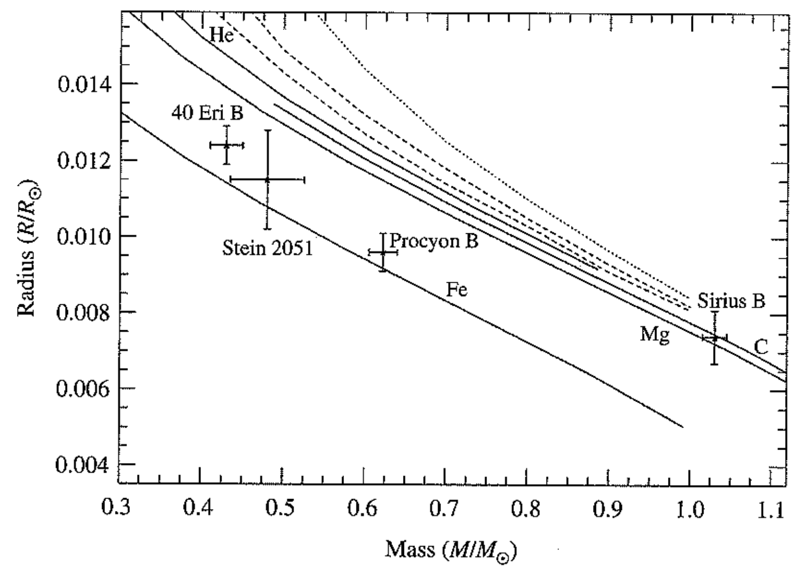


FIGURE 18.1 Observed masses and radii for nearby white dwarfs. The solid and dashed lines represent models for different chemical compositions.

The measured masses and radii of some nearby white dwarfs are plotted in Figure 18.1. The best-determined masses and radii are for Sirius B, Procyon B, and 40 Eri B, all part of visual binary systems.

As the pressure P drops with distance from the center of the white dwarf, so does the density $\rho \propto P^{3/5}$ (equation 18.13). The outermost layers of a white dwarf are thus low enough in density for ordinary thermal pressure to be the dominant pressure source. The nondegenerate atmosphere of the white dwarf is different in some ways from the atmospheres of ordinary stars. The gravitational acceleration in the white dwarf's atmosphere is large:

$$g_{wd} = \frac{GM_{wd}}{R_{wd}^2} \approx 7000 g_\odot \left(\frac{M}{0.7M_\odot}\right)^{5/3} \approx 2 \times 10^6\ \mathrm{m\ s^{-2}} \left(\frac{M}{0.7M_\odot}\right)^{5/3}. \qquad (18.18)$$

Thus, a white dwarf's atmosphere will have a small scale height, despite the high photospheric temperature ($T \sim 10^5$ K $\sim 20T_\odot$) of a newly unveiled white dwarf:

$$H_{wd} = \frac{kT_{wd}}{g_{wd}\mu_{wd} m_p} \sim 0.4\ \mathrm{km} \left(\frac{T_{wd}}{10^5\ \mathrm{K}}\right)\left(\frac{M_{wd}}{0.7M_\odot}\right)^{-5/3}. \qquad (18.19)$$

Because of the high gravitational acceleration in the photosphere, the spectra of white dwarfs typically show extreme pressure broadening of absorption lines (Figure 18.2). Thus, white dwarfs can be distinguished from hot main sequence stars by their spectra alone.

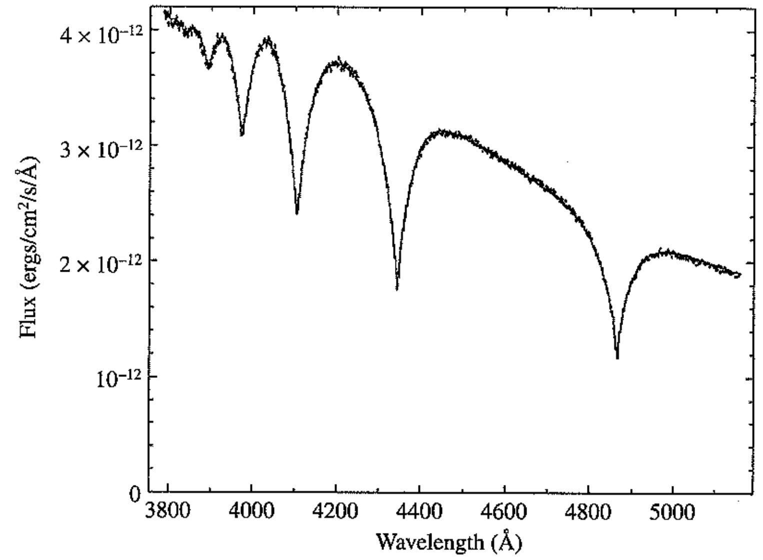


FIGURE 18.2 Spectrum of the white dwarf Sirius B; note the pressure-broadened Balmer lines.

Sirius B, with a mass equal to that of the Sun, is the most massive white dwarf in our immediate neighborhood (see Figure 18.1). White dwarfs cannot have an arbitrarily high mass. As additional mass is piled on to a white dwarf, the density increases, and the "Heisenberg speed" of the electrons,

$$\Delta v \sim \hbar \frac{n_e^{1/3}}{m_e}, \tag{18.20}$$

approaches the speed of light. When $\Delta v \sim c$, rearranging equation (18.20) gives

$$n_e \sim \left(\frac{cm_e}{\hbar}\right)^3 \sim 2 \times 10^{37}\,\mathrm{m}^{-3}, \tag{18.21}$$

so the degenerate electrons become relativistic at this density. Since a typical white dwarf has one proton and one neutron for each electron, this corresponds to a mass density

$$\rho \sim 2m_p n_e \sim \frac{2c^3 m_e^3 m_p}{\hbar^3} \sim 6 \times 10^{10}\,\mathrm{kg\,m}^{-3}. \tag{18.22}$$

When we compare this critical density to the mass–density relation for white dwarfs (equation 18.16), we find that the degenerate electrons in a white dwarf become relativistic when

$$\frac{G^3 m_e^3 m_p^5}{\hbar^6} M^2 \sim \frac{c^3 m_e^3 m_p}{\hbar^3}, \tag{18.23}$$

or when the white dwarf's mass is

$$M \sim \left(\frac{\hbar^3 c^3}{G^3 m_p^4}\right)^{1/2} \sim 4 \times 10^{30}\,\mathrm{kg}, \tag{18.24}$$

or about twice the mass of the Sun.

The transition of the degenerate electrons from nonrelativistic to relativistic has grave consequences for the structure of the white dwarf. Highly relativistic electrons have an energy much greater than their rest energy: $E_{\rm rel} \gg m_e c^2$. In practice, we can treat them like massless particles (photons, for instance). For instance, photons have an energy related to their momentum by the equation $E = pc$. Similarly, highly relativistic degenerate electrons will each have an energy

$$E_{\rm rel} \sim (\Delta p)c \sim \hbar n_e^{1/3} c, \tag{18.25}$$

providing a total electron energy density

$$u_{\rm rel} \equiv E_{\rm rel} n_e \sim \hbar c n_e^{4/3}. \tag{18.26}$$

By analogy, once again, with photons, which have a radiation pressure $P = u/3$, we can compute the pressure contributed by the highly relativistic degenerate electrons:

$$P_{\rm rel} = \frac{1}{3} u_{\rm rel} \sim \frac{\hbar c}{3} n_e^{4/3}. \tag{18.27}$$

Note the different dependence on n_e than that which held true for *nonrelativistic* electrons, which had $P \propto n_e^{5/3}$. The pressure in the relativistic case is less strongly dependent on density.[4]

If the relativistic white dwarf is to remain in hydrostatic equilibrium (equation 18.3), it must have

$$P_c \sim G\frac{M^2}{R^4}. \tag{18.28}$$

The pressure provided by the relativistic degenerate electrons is

$$P_{c,\rm rel} \sim \hbar c \left(\frac{\rho}{m_p}\right)^{4/3} \sim \frac{\hbar c}{m_p^{4/3}} \frac{M^{4/3}}{R^4}. \tag{18.29}$$

Setting these two pressures equal, we find that a white dwarf supported by relativistic degenerate electrons will be in equilibrium when

$$G\frac{M^2}{R^4} \sim \frac{\hbar c}{m_p^{4/3}} \frac{M^{4/3}}{R^4}. \tag{18.30}$$

[4] In other words, the relativistic white dwarf isn't as stiff; a stiff material is one in which a small change in density produces a large change in pressure.

Note that the factors of R^{-4} cancel on either side. This means that a relativistic white dwarf is only in equilibrium for a specific mass,

$$M \sim \left(\frac{\hbar^3 c^3}{G^3 m_p^4}\right)^{1/2}. \tag{18.31}$$

But this is just the mass that we computed in equation (18.24) as the minimum mass required for the electrons to be relativistic! For any mass greater than this value, $M \sim 2M_\odot$, the central pressure required for hydrostatic equilibrium is *greater* than the pressure that relativistic degenerate electrons can supply, and the white dwarf collapses.

The maximum possible mass for a white dwarf, $M_{Ch} \sim 2M_\odot$, is called the **Chandrasekhar mass**, after the astrophysicist Subramanyan Chandrasekhar, who was the first to calculate it. A more careful calculation of the Chandrasekhar mass for a carbon/oxygen white dwarf yields

$$M_{Ch} = 1.4M_\odot. \tag{18.32}$$

Any star that can reduce its mass below the Chandrasekhar mass (generally by strong stellar winds during a giant phase) will end as a white dwarf. It is estimated that stars with $M \leq 7M_\odot$ will be able to slim themselves down below the Chandrasekhar mass. This calculation is uncertain, though, since mass loss during the giant phase is irregular by nature and difficult to model. Stars with initial masses less than $0.5M_\odot$ will eventually become helium white dwarfs; stars with $0.5M_\odot < M < 5M_\odot$ initially will leave behind carbon/oxygen white dwarfs; stars with $5M_\odot < M < 7M_\odot$ will leave behind neon/magnesium white dwarfs.

18.2 ▪ NEUTRON STARS AND PULSARS

What happens to stars whose initial mass is greater than $7M_\odot$? There aren't many stars that massive, but they do exist. Very massive stars have short lives and die spectacularly, leaving behind a badly crushed corpse. The main sequence lifetime of a massive star is short:

$$\tau \approx 30\,\text{Myr}\left(\frac{M}{7M_\odot}\right)^{-3}. \tag{18.33}$$

Life after the main sequence, when the star becomes a supergiant, is even shorter. The ultraluminous supergiant is relying on less efficient energy sources. Fusing hydrogen to helium releases about $6.4 \times 10^{14}\,\text{J kg}^{-1}$; thereafter, fusing helium all the way to iron releases only 24% as much energy per kilogram. In its last moments as a star, a massive supergiant has many concentric fusion layers over an iron core. A schematic diagram of such a star is shown in Figure 18.3.

As we saw in Section 13.4, a supergiant star such as Betelgeuse can have a radius $> 1000R_\odot$; however, models of stellar evolution indicate that the dense central fusion region of the supergiant has a radius $< 1R_\odot$. The central iron core is very dense (thou-

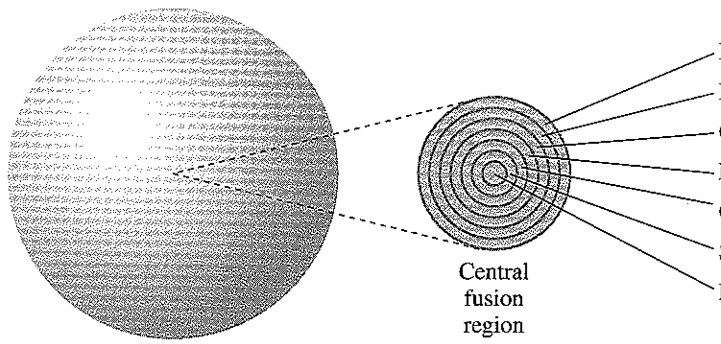


FIGURE 18.3 Fusion layers in a supergiant near the end of its life as a star.

sands of tons per cubic centimeter) and is supported by degenerate electron pressure. The iron core continues to grow in size as the silicon-fusing shell eats its way outward in the supergiant. When the iron core reaches the Chandrasekhar mass, it is no longer adequately supported by the relativistic degenerate electrons, and it starts to collapse—very rapidly. The collapse time t_{ff} at the high density of a degenerate core is less than 1/10 second.

As the density of the collapsing iron core rises, protons and free electrons start to combine to form neutrons:

$$p + e^- \rightarrow n + \nu_e. \tag{18.34}$$

One side effect of the collapse is thus a huge burst of electron neutrinos (ν_e). A Chandrasekhar mass of iron contains $\sim 10^{57}$ protons, so $\sim 10^{57}$ neutrinos are created during the collapse of the iron core. This burst of neutrinos escapes from the star, carrying away a large amount of energy, about 10^{46} J. (For comparison, the Sun will radiate $\sim 10^{44}$ J of energy during its main sequence lifetime.) The core is now a sphere of $\sim 2 \times 10^{57}$ neutrons that are essentially in free fall. Can anything stop the headlong collapse of the neutrons?

Yes! Core collapse can be stopped by **neutron degeneracy pressure**. Degenerate nonrelativistic electrons have a pressure

$$P_e \sim \hbar \frac{n_e^{5/3}}{m_e}. \tag{18.35}$$

The Fermi exclusion principle applies to neutrons as well as to electrons. Degenerate nonrelativistic neutrons thus have a pressure

$$P_n \sim \hbar \frac{n_n^{5/3}}{m_n}, \tag{18.36}$$

where n_n is the number density of neutrons and $m_n = 1839m_e$ is the mass of a neutron. A sphere of neutrons supported by degenerate neutron pressure is called a **neutron star**.

(It doesn't satisfy our strict definition of a "star," since a neutron star isn't gaseous and isn't powered by fusion, but the name has stuck.)

A white dwarf—a sphere supported by electron degeneracy pressure—has a radius (equation 18.15)

$$R_{\rm wd} \sim \frac{\hbar^2}{G m_e m_p^2}\left(\frac{M}{m_p}\right)^{-1/3}. \qquad (18.37)$$

By analogy, a neutron star—a sphere supported by neutron degeneracy pressure—should have a radius

$$R_{\rm ns} \sim \frac{\hbar^2}{G m_n m_p^2}\left(\frac{M}{m_p}\right)^{-1/3}. \qquad (18.38)$$

A neutron star will thus be smaller than a white dwarf of comparable mass, by a ratio

$$\frac{R_{\rm ns}}{R_{\rm wd}} \sim \frac{m_e}{m_n}\left(\frac{M_{\rm wd}}{M_{\rm ns}}\right)^{1/3} \sim \frac{1}{1839}\left(\frac{M_{\rm wd}}{M_{\rm ns}}\right)^{1/3}. \qquad (18.39)$$

If a white dwarf with $M_{\rm wd} = 0.7M_\odot$ has a radius $R_{\rm wd} = 0.01R_\odot$, then equation (18.39) leads us to expect

$$R_{\rm ns} \sim 3\,{\rm km}\left(\frac{M_{\rm ns}}{1.4M_\odot}\right)^{-1/3}. \qquad (18.40)$$

In truth, things are a little more complicated than we've been letting on. The density within a neutron star is comparable to the density within an atomic nucleus. Thus, the strong nuclear force between neutrons must be taken into account. Although the strong nuclear force is attractive at distances $d > 5 \times 10^{-16}$ m (this attraction is what keeps nuclei from flying apart), it is repulsive at shorter distances. Thus, the short-range repulsion between neutrons stiffens the neutron star and keeps the neutrons from being shoved arbitrarily close together.

For one plausible model of neutron star interiors, which takes the strong nuclear force into account, the mass–radius relation is

$$R_{\rm ns} \approx 11\,{\rm km}\left(\frac{M_{\rm ns}}{1.4M_\odot}\right)^{-1/3}. \qquad (18.41)$$

White dwarfs contain the mass of a star squashed into the volume of the Earth; neutron stars contain the mass of a star squashed into the volume of a small asteroid. It is a cliché for an astronomy text to show a neutron star juxtaposed with a city (Figure 18.4). Nevertheless, there's a reason why so many books include such a figure; it's an effective way of showing how tiny neutron stars are for their mass.

Understanding the structure of neutron stars is difficult for two reasons. Not only must the strong nuclear force be taken into account, but also gravity must be treated using general relativity rather than Newtonian gravity. The escape speed from a neutron

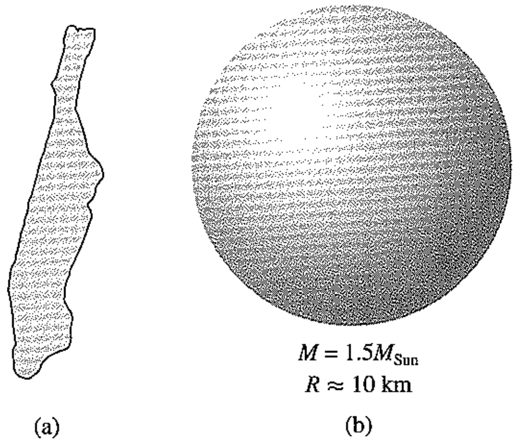


FIGURE 18.4 Manhattan (a) compared with a neutron star (b).

star, using the Newtonian formula (equation 3.62), is

$$v_{\rm esc} = \left(\frac{2GM_{\rm ns}}{R_{\rm ns}}\right)^{1/2} \approx 2\times 10^8\,{\rm m\,s^{-1}}\left(\frac{M_{\rm ns}}{1.4M_\odot}\right)^{2/3}. \qquad (18.42)$$

The escape speed from a neutron star is a significant fraction of the speed of light:

$$v_{\rm esc}/c \approx 0.6\left(\frac{M_{\rm ns}}{1.4M_\odot}\right)^{2/3}. \qquad (18.43)$$

If we crave accuracy, we really should be using general relativity in the vicinity of a neutron star, rather than classical Newtonian dynamics.

Since neutron stars are tiny, you might think they'd be difficult to detect in the vast darkness of interstellar space. In fact, there's more than one way to detect a neutron star. When neutron stars are first formed, they have hot surfaces, with $T_{\rm ns} \approx 10^6\,{\rm K} \approx 170T_\odot$. The radius of the neutron star is small, with $R_{\rm ns} \approx 11\,{\rm km} \approx 1.6\times 10^{-5}R_\odot$. The blackbody luminosity of the neutron star is then

$$L_{\rm ns} = \left(\frac{R_{\rm ns}}{R_\odot}\right)^2\left(\frac{T_{\rm ns}}{T_\odot}\right)^4 L_\odot \approx (1.6\times 10^{-5})^2(170)^4 L_\odot \approx 0.2L_\odot. \qquad (18.44)$$

A neutron star thus has a respectable luminosity. Its wavelength of maximum emission, given $T_{\rm ns} \approx 10^6$ K, is $\lambda_p \approx 30$ Å, corresponding to a photon energy of $E \sim 400$ eV in the X-ray range of the spectrum. Thus, isolated neutron stars can be detected by X-ray satellites. Figure 18.5 shows the spectrum of a relatively nearby neutron star ($d \sim 120$ pc) observed by the *Chandra X-ray Observatory*.

Neutron stars are capable of creating photons in ways other than simple blackbody emission. Because angular momentum is conserved during core collapse, neutron stars

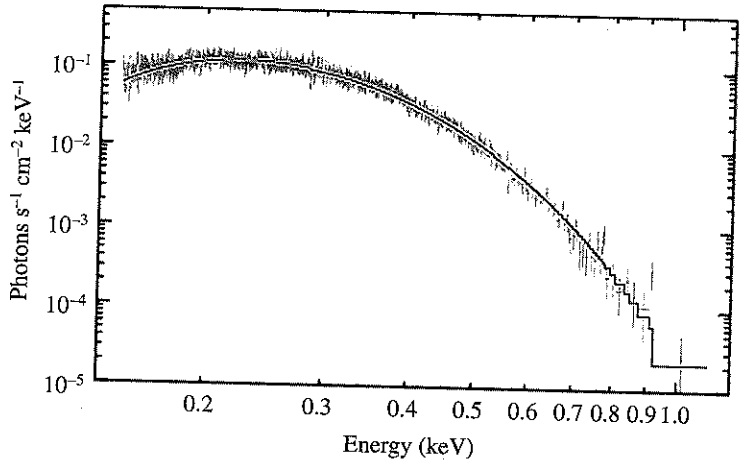


FIGURE 18.5 X-ray spectrum of the neutron star RX J1856.5-3754. The superimposed solid line represents the spectrum of a blackbody with $kT = 63$ eV.

rotate rapidly. Rotation speeds as great as $v \sim 0.1c$ are possible, with a corresponding rotation period of

$$P_{ns} \sim \frac{2\pi R_{ns}}{0.1c} \sim 2 \times 10^{-3}\,\text{s}. \tag{18.45}$$

Because the magnetic flux threading through the core is also conserved during collapse, neutron stars have strong magnetic fields. Compared to the magnetic field strength of $B_\odot \approx 10^{-4}$ Tesla at the Sun's surface, a neutron star can have $B_{ns} \approx 10^6$ Tesla. Rapidly rotating, highly magnetized neutron stars are **pulsars**. The name "pulsar" was given not because they pulsate in and out like Cepheids (they don't!) but because they produce pulses of electromagnetic radiation as seen from Earth. Pulsars were first detected in 1967, during a radio survey of the sky. No one knew what they were at first.[5] It was soon realized, however, that they must be neutron stars. Some pulsars have periods as short as a millisecond, but others have gradually spun down to periods of more than a second.

A neutron star has a surface gravitational acceleration of

$$g_{ns} = \frac{GM_{ns}}{R_{ns}^2} \sim 6 \times 10^9 g_\odot \sim 1.5 \times 10^{12}\,\text{m s}^{-2}. \tag{18.46}$$

The atmosphere of the neutron star consists of ionized gas supported by ordinary thermal pressure, with a scale height

$$H_{ns} = \frac{kT_{ns}}{g_{ns}\mu m_p} \sim 0.3\,\text{cm}. \tag{18.47}$$

[5] The first pulsar discovered was given the half-joking name "LGM-1," standing for Little Green Men.

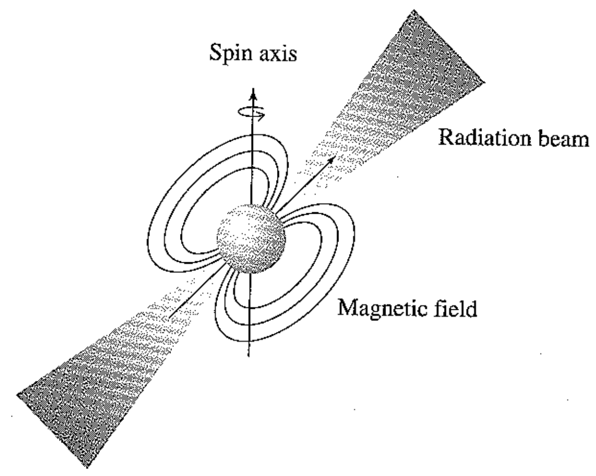


FIGURE 18.6 "Lighthouse model" of a pulsar.

The rotating magnetic field of the neutron star creates a strong electric field. This field is capable of ripping electrons and ions out of the neutron star's atmosphere and accelerating them along magnetic field lines, where they produce copious synchrotron radiation. The synchrotron emission tends to be beamed along the two magnetic axes of the neutron star (Figure 18.6). Since the magnetic axis isn't aligned perfectly with the rotation axis, the beams of synchrotron emission sweep around and around as the neutron star rotates. Neutron stars whose beams of synchrotron emission happen to sweep across the Earth's location are labeled "pulsars" by Earthlings, since we detect periodic pulses of synchrotron radiation from them. This is similar to the way in which we see pulses of light from a lighthouse whenever its rotating beams of light sweep across our location. There are about 500 pulsars known to us. However, since the synchrotron beams of pulsars are fairly narrow, we expect the number of pulsars (as seen from Earth) to be significantly smaller than the number of neutron stars.

Why are we sure that pulsars are rotating magnetized neutron stars? For one thing, we don't know of anything else that would produce strong pulses of light at such short periods. If a pulsar were actually undergoing radial pulsations with a period $P = 10^{-3}$ s, its density would have to be (see Section 17.3)

$$\langle \rho \rangle \sim \frac{1}{GP^2} \sim 10^{16}\,\text{kg m}^{-3}, \tag{18.48}$$

denser than any known white dwarf. If a pulsar were a rotating white dwarf, it would have to rotate with a speed

$$v = \frac{2\pi r}{P} \sim 100c \left(\frac{M}{0.7M_\odot}\right)^{-1/3}. \tag{18.49}$$

A clinching piece of circumstantial evidence is that some pulsars are found within **supernova remnants**. Both neutron stars and supernova remnants are created as the consequence of core collapse in a massive star.

Let's briefly review events during core collapse:

- The iron core of a massive star grows larger than the Chandrasekhar mass. No longer supported by degenerate electron pressure, the core starts to collapse.
- Protons and electrons combine to form neutrons, and 10^{57} neutrinos are produced. The collapsing neutron sphere is actually dense enough to be opaque to neutrinos, so it takes them a while to work their way out in a random walk.
- The neutron sphere is compressed slightly beyond the point where degenerate neutron pressure balances gravity, and bounces back.
- The "core bounce" sends a shock wave through the outer layers of the star. After 30 milliseconds or so, the amount of matter swept up by the shock wave is sufficient to temporarily stall the shock's outward motion.
- The neutrinos making their way out of the neutron star interact with the dense gas in the shock wave, heating the gas and causing the shock wave to start moving outward again.
- The outer layers are then ejected at high speed (up to $\sim 15{,}000\ \mathrm{km\ s^{-1}}$, or $\sim 0.05c$).

One interesting side effect of the shock wave is that it produces small amounts of elements heavier than iron in the shock wave. (It takes energy to create these ultramassive elements, but the supernova has energy to spare.) Another interesting side effect is that the expanding gas becomes hot and emits lots of light.

The light emitted by the shock-heated gas is what we usually think of when we talk about a **supernova** (discussed further in Section 18.4). However, photons carry only a small amount of the energy associated with core collapse. Here's a more complete accounting:

- Neutrino energy $\sim 10^{46}$ J
- Kinetic energy of ejected gas $\sim 10^{44}$ J
- Photon energy $\sim 10^{42}$ J

This energy comes from the gravitational potential energy of the pre-collapse core. A neutron star has gravitational potential energy

$$U_{ns} \approx -\frac{3}{5}\frac{GM_{ns}^2}{R_{ns}} \approx -3 \times 10^{46}\ \mathrm{J}. \qquad (18.50)$$

This is about 10% of the rest energy of a neutron star, $M_{ns}c^2 \approx 3 \times 10^{47}$ J; thus, compressing matter into a neutron star is a fairly efficient way of converting its mass into energy.

At maximum luminosity, the supernova emits photons with a luminosity $L_{sn} \sim 10^9 L_\odot$. However, the supernova decreases in luminosity by a factor of 100 over a

FIGURE 18.7 Wall painting in Chaco Canyon, New Mexico, thought to depict the 1054 supernova next to the crescent Moon.

few months; supernovae are bright but brief-lived phenomena. An interesting tidbit of information is that the nearest star with $M > 7M_\odot$ is Betelgeuse, at $d \approx 130$ pc. When it becomes a supernova (any millennium, now!), its flux as seen from Earth will be comparable to that of the full Moon. There haven't been any Betelgeuse-scale supernovae during recorded history, but there have been some comparable in flux to Venus. On AD 1054 July 5 (in the Julian calendar), a supernova appeared in the constellation Taurus. Chinese astronomical records report that the "guest star" had a flux corresponding to $m \sim -4$ in the system of Hipparchus. The supernova was also commemorated in an Anasazi wall painting (Figure 18.7).

Today, nearly 1000 years later, when we turn our telescopes to the position recorded by the Chinese astronomers, we see the Crab Nebula, a supernova remnant (Color Figure 18). The Crab Nebula is 1.5 pc in radius, and its expansion speed, measured from Doppler shifts, is $1500\ \mathrm{km\ s^{-1}}$. A little calculation soon shows that $1.5\ \mathrm{pc} \approx (1500\ \mathrm{km\ s^{-1}})(1000\ \mathrm{yr})$. The Crab Nebula shows all the signs of being the expanding cloud of debris from an explosion about 1000 years earlier. In the center of the Crab Nebula is a pulsar. The Crab pulsar has a period of 33 milliseconds and is seen to pulsate at radio, visible, X-ray, and gamma-ray wavelengths.[6] The pulsar is in the Crab Nebula, just where you'd expect the neutron star; it's a classic "smoking gun" piece of evidence.

18.3 ▪ BLACK HOLES

Neutron stars are not the ultimate in compression for stellar remnants. There exists an upper limit to neutron star masses, the **Oppenheimer–Volkov limit**, that is analogous to the Chandrasekhar mass for white dwarfs. The upper mass limit for neutron stars is

[6] Curiously, astronomers had been observing the pulsar for years at visible wavelengths without noticing the strobe effect—the pulses were too fast.

difficult to calculate, because the strong nuclear force must be taken into account and gravity must be treated using general relativity. Although there is still some debate among neutron star mavens, an upper mass limit of

$$M_{\max} \approx 3M_\odot \tag{18.51}$$

for neutron stars is generally accepted. Stars with initial mass $M > 18M_\odot$ or so will leave behind remnants with $M > M_{\max}$. These ultramassive, ultraluminous, short-lived stars will leave behind **black holes** as their remnants.

A black hole can be defined, quite simply, as an object whose escape speed is greater than the speed of light. For a spherical body,

$$v_{\rm esc} = \left(\frac{2GM}{r}\right)^{1/2}, \tag{18.52}$$

so $v_{\rm esc} = c$ when a body of mass M has a radius

$$r = \frac{2GM}{c^2} \equiv r_{\rm Sch}. \tag{18.53}$$

The critical radius at which a mass M has an escape speed equal to the speed of light is called the **Schwarzschild radius,** $r_{\rm Sch}$, after the physicist Karl Schwarzschild, who first calculated it in a relativistically correct manner. (In general, you don't expect Newtonian calculations to give the correct results in the highly relativistic regime. In the case of computing the Schwarzschild radius, however, it works out correctly.)

Any object will become a black hole if you squeeze it until it is smaller than its Schwarzschild radius. An astronomer with $M \approx 70$ kg will become a black hole if he/she is squeezed to a radius $r_{\rm Sch} \approx 10^{-25}$ m. While it's not practical (nor in most cases desirable) to squeeze an astronomer to this submicroscopic size, it is practical to squeeze an extremely massive star down to its Schwarzschild radius. Just let gravity do the work. For a massive stellar remnant,

$$r_{\rm Sch} = \frac{2GM}{c^2} = 3\,{\rm km}\left(\frac{M}{M_\odot}\right). \tag{18.54}$$

Every black hole is surrounded by an **event horizon**: a spherical surface whose circumference is equal to 2π times the Schwarzschild radius. It is possible to enter the event horizon, but it is not possible to emerge again. Nothing, not even light, travels fast enough to escape from inside the event horizon.

General relativity predicts the existence of singularities within event horizons. A singularity is a point of infinite density and infinite spacetime curvature. However, to test the predictions of general relativity, and see whether singularities really exist, you'd have to enter the event horizon of a black hole. Presumably, once there, you'd be able to discover the answer, but you wouldn't be able to communicate your results to the outside world. It would be the ultimate scientific tragedy: having a great result but being unable to publish it.

Lurid sci-fi movies sometimes regard black holes as dangerous "vacuum cleaners," sucking up everything within reach. In fact, when you are far outside the Schwarzschild

radius of a black hole, its gravitational pull is just the same as you'd feel from any other object of the same mass. What makes black holes potentially dangerous is their extremely compact nature; as you approach the central singularity of a black hole, you feel stronger and stronger tidal forces, until you are eventually ripped apart. If you want to spare your friends and relatives the gory sight of your demise, you should take care to dive toward a high-mass black hole rather than a low-mass black hole. For a low-mass black hole, you will be ripped apart *before* entering the event horizon; for a high-mass black hole, you will be ripped apart *after* entering the event horizon. To see why this is so, let's review what happens as you drop (feet first) toward a black hole.

The tidal force pulling you apart will be, from Section 4.2,

$$\Delta F \approx \frac{GMm}{r^3}\ell. \tag{18.55}$$

Here m is your mass and ℓ is your height; if you're an average sort of person, $m \sim 70$ kg and $\ell \sim 1.8$ m, respectively. The black hole's mass is M, and your distance from the black hole is r. You will be torn apart when the tidal force reaches a critical value $F_{\rm rip}$. The classic studies of M. Python reveal that the force exerted by a 16-ton weight is adequate to crush a human being.[7] Since the human body is about as strong in extension as in compression, let's take $F_{\rm rip} = 16$ tons $= 32{,}000$ lb $= 1.4 \times 10^5$ N. Thus, the radius at which you'll be ripped apart is

$$r_{\rm rip} = \left(\frac{GMm\ell}{F_{\rm rip}}\right)^{1/3} \approx 480\,{\rm km}\left(\frac{M}{1M_\odot}\right)^{1/3}. \tag{18.56}$$

Since the Schwarzschild radius of a black hole is

$$r_{\rm Sch} = 3\,{\rm km}\left(\frac{M}{1M_\odot}\right), \tag{18.57}$$

the ratio of the "ripping radius" to the Schwarzschild radius is

$$\frac{r_{\rm rip}}{r_{\rm Sch}} \approx 160\left(\frac{M}{1M_\odot}\right)^{-2/3}. \tag{18.58}$$

You will be ripped apart exactly at the Schwarzschild radius when $M \approx 2000M_\odot$. For lower-mass black holes, $r_{\rm rip} > r_{\rm Sch}$, so you will be torn apart before having a chance to reach the event horizon. If you want to see what life is like inside an event horizon, be sure to choose a black hole with $M \gg 2000M_\odot$. (You can curl yourself into a sphere to decrease ℓ, but this delays your tidal disruption by only a small amount.)

[7] See, for example, *Monty Python's Flying Circus,* Season 1, Episode 4, "Self-defence Against Men Armed with Fruit" sketch. If more recent experiments on the compressional strength of the human body have been performed, the authors would greatly appreciate *not* hearing about them.

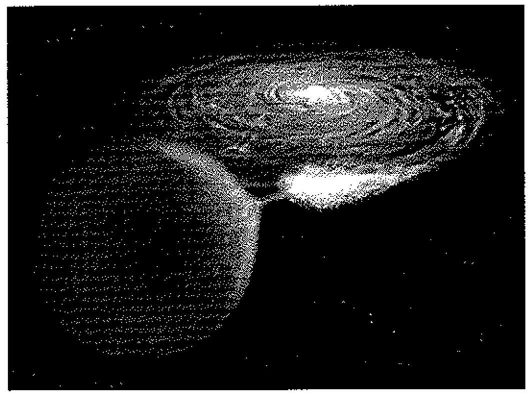


FIGURE 18.8 Artist's impression of a black hole accreting gas from a stellar companion.

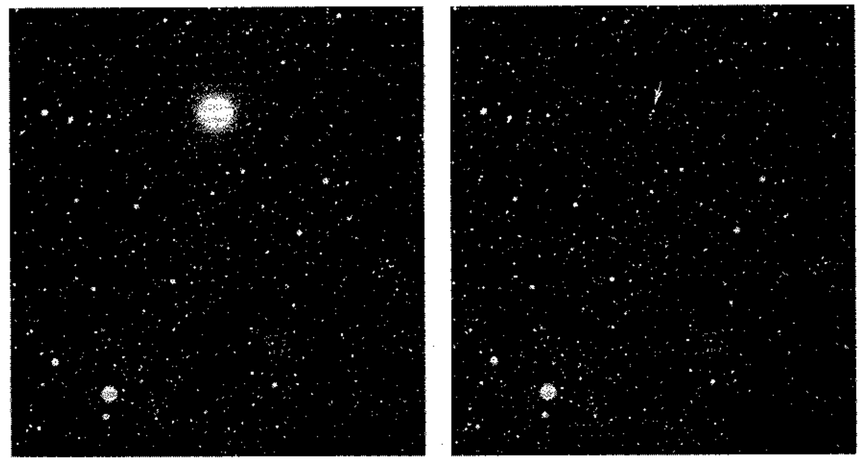
If you want to dive into a black hole, you must first find a black hole. This can be difficult, since (after all) a black hole is black.[8] We can, however, detect black holes indirectly, by their gravitational effects on nearby matter. Consider, for instance, a black hole that is in a binary system with a normal star (Figure 18.8). If the normal star, as it evolves, swells until it is larger than its Roche limit (see Section 4.3.1), the tidal force exerted by the black hole will be great enough to strip away the outer layers of the star. The stripped gas, as it falls toward the black hole, will be compressed and heated to $\sim 10^6$ K or so. Since the falling gas will have some angular momentum, it will generally form an **accretion disk** around the black hole, as shown in Figure 18.8. The disk of hot gas will be detectable from the X-rays it emits.

One strong candidate for a binary system including a black hole is the X-ray source V404 Cygni. At visible wavelengths, V404 Cygni is a spectroscopic binary, with a relatively cool subgiant (spectral type K0 IV) orbiting a dark massive companion with a period of $P = 6.47$ days. If we are looking at the orbit of the subgiant edge-on, the orbital speed we deduce from its periodically varying Doppler shift is $v_c = 209\ \mathrm{km\ s^{-1}}$. The minimum possible mass for the dark massive companion, given these observed parameters, is $M_{\rm bh} = 6.1 M_\odot$, far higher than the maximum permissible mass for a neutron star. A more detailed model of the V404 Cygni system assigns a mass of $M_{\rm bh} = 10 M_\odot$ to the black hole and of $M_{\rm star} = 0.6 M_\odot$ to the subgiant star.

18.4 ▪ NOVAE AND SUPERNOVAE

On the evening of 1572 November 11, Tycho Brahe was contemplating the night sky when he saw what he thought was a new star in the constellation Cassiopeia. Since this

[8] The quantum effect known as Hawking radiation, which causes particles and antiparticles to be emitted from the region near the event horizon, is significant only for small black hole masses. A black hole with $M = 3M_\odot$ has a luminosity in Hawking radiation of only $L = 3 \times 10^{-22}$ W.



(a) (b)

FIGURE 18.9 Nova V1500 Cygni, during its outburst (a) and after its outburst (b), when it had dimmed to $m_v \sim 15$.

was a blow to the Aristotelian dogma that no new stars can appear in the heavens, Tycho rushed into print with his new work *De Nova Stella* ("On the New Star").[9] Following the lead of Tycho, astronomers applied the term "nova" to unresolved celestial objects whose flux increases by a large amount over a short period of time, giving the impression of a new star appearing in the sky.

When a nova produces an outburst of light, its luminosity can briefly increase by a very large factor. For example, consider the nova V1500 Cygni, seen in the year 1975 (Figure 18.9). Before its outburst, V1500 Cygni was an inconspicuous object, with $m_V \approx 20$. At the peak of its outburst, it had $m_V \approx 2$, making it the second brightest source in Cygnus, after the star Deneb. At maximum, therefore, the flux of V1500 Cygni had increased by a factor $10^{0.4(20-2)} \approx 2 \times 10^6$. The sharp rise to maximum flux was followed by a gradual decline over the course of months.

The modern definition of **nova** states that a nova is a type of cataclysmic variable (as opposed to a pulsating variable like a Cepheid) in which a normal star dumps gas onto a white dwarf. Thus, a nova is actually a close binary system in which a star and a white dwarf orbit their mutual center of mass. Tossing material onto a compact object such as a white dwarf or neutron star is an excellent way to convert gravitational potential energy into thermal energy, and then into photons. However, we still need to explain why pouring gas onto a white dwarf produces sudden cataclysmic outbursts of light, rather than a steady glow.

Let's look at what happens when a star expands past its Roche limit and pours gaseous hydrogen onto its close companion, a white dwarf. (The outer layers of a star are actually

[9] More fully, *De Nova et Nullius Aevi Memoria Prius Visa Stella*, or "On the New and Never Previously Seen Star."