Chapter 1

Introduction

The world of our every-day experiences – the world of the not too big (compared to, say, a galaxy), and the not too small, (compared to something the size and mass of an atom), and where nothing moves too fast (compared to the speed of light) – is the world that is mostly directly accessible to our senses. This is the world usually more than adequately described by the theories of classical physics that dominated the nineteenth century: Newton's laws of motion, including his law of gravitation, Maxwell's equations for the electromagnetic field, the three laws of thermodynamics. These classical theories are characterized by, amongst other things, the notion that there is a 'real' world out there, one that has an existence independent of ourselves, in which, for instance, objects have a definite position and momentum which we could measure to any degree of accuracy, limited only by our experimental ingenuity. According to this view, the universe is evolving in a way completely determined by these classical laws, so that if it were possible to measure the position and momenta of all the constituent particles of the universe, and we knew all the forces that acted between the particles, then we could in principle predict to what ever degree of accuracy we desire, exactly how the universe (including ourselves) will evolve. Everything is predetermined – there is no such thing as free will, there is no room for chance. Anything apparently random only appears that way because of our ignorance of all the information that we would need to have to be able to make precise predictions.

This rather gloomy view of the nature of our world did not survive long into the twentieth century which saw the formulation of a new set of fundamental principles that provides a framework into which all physical theories must fit, and to that extent all natural phenomena are governed to a greater or lesser extent by its laws: quantum mechanics. One of the crucial features of quantum mechanics is the loss of determinancy: irreducible randomness is built into the laws of nature. The world is inherently probabilistic in that events can happen without a cause, a fact first stumbled on by Einstein, but never fully accepted by him.

Quantum mechanics is often thought of as being the physics of the very small, but this is true only insofar as the fact that peculiarly quantum effects are most readily observed at the atomic level. But in the everyday world that we usually experience, where the classical laws of Newton and Maxwell seem to be able to explain so much, it quickly becomes apparent that classical theory is unable to explain many things e.g. why a solid is 'solid', or why a hot object has the colour that it does. Beyond that, quantum mechanics is needed to explain radioactivity, the chemical properties of matter, how semiconducting devices work, superconductivity, the interaction between light and matter (leading to describing what makes a laser do what it does), the properties of elementary particles such as quarks, muons, neutrinos, ... Even on the very large scale, quantum effects leave their mark in unexpected ways: the galaxies spread throughout the universe are believed to be macroscopic manifestations of microscopic quantum-induced inhomogeneities present shortly after the birth of the universe, when the universe itself was tinier than an atomic nucleus and almost wholly quantum mechanical. Indeed, the marriage of quantum mechanics – the physics of the very small – with general relativity – the physics of the very large – is believed by some to be the crucial step in formulating a general 'theory of everything' – superstring theory – that will hopefully contain all the basic laws of nature in one package. The impact of quantum mechanics on our view of the world and the natural laws that govern it, cannot be underestimated. But the subject is not entirely esoteric. Its consequences have been exploited in many ways that have an immediate impact on the quality of our lives. It has been estimated that the economical impact of quantum mechanics cannot be ignored: about 30% of the gross national product of the United States is based on inventions made possible by quantum mechanics. If anyone aims to have anything like a broad understanding of the sciences that underpin modern technology, as well as obtaining some insight into the modern view of the character of the physical world, then some knowledge and understanding of quantum mechanics is mandatory.

As we have just seen, quantum mechanics is essential in providing a framework of physical and mathematical principles with which we attempt to understand the physical nature of the world in which we live. Its success in doing just that has been extraordinary. Yet for all of that, and in spite of the fact that the theory is now roughly 100 years old, if Planck's theory of black body radiation is taken as being the birth of quantum mechanics, it as true now as it was then that no one truly understands the theory, though in recent times, a greater awareness has developed of what quantum mechanics is all about: it is a theory of information, that is, it is a theory concerning what information we can gain about the world about us – nature places limitations on what we can 'know' about the physical world, but it also gives us greater freedoms concerning what we can do with this 'quantum information' (as compared to what we could expect classically), as realized by recent developments in quantum computation, quantum teleportation, quantum cryptography and so on. In any case, following the principles of quantum mechanics, it is possible to provide an explanation of everything from the state of the universe immediately after the big bang, to the structure of DNA, to the colour of your socks.

The most familiar version of the quantum theory goes by the name 'wave mechanics', and its most familiar application is to describing the structure of matter at the atomic level: atomic, molecular and solid state physics, built as it is around the wave function ψ and the interpretation of $|\psi|^2$ as giving the probability of finding a particle in some region in space. But quantum mechanics is much more than the mechanics of the wave function, and its applicability goes way beyond atomic, molecular or solid state theory. Quantum mechanics is a set of fundamental principles that presumably apply to all physical systems: to the electromagnetic field and the other force fields of nature, to quarks and electrons and other fundamental particles, which can be created or destroyed and which possess such properties as spin, charge, colour, flavour, to many particle systems such as the electrons in a metal, or photons in a laser beam. These principles embody fundamental physical and philosophical issues that can be abstracted from and studied independently of any physical system that could potentially display them. It is these principles abstracted in this way that constitute quantum mechanics. To describe the quantum properties of such a wide variety of physical phenomena, and to provide a language that contains all of the basic principles of quantum mechanics without being tied to the notion of the wave function, a more general perspective is required.

A deeper look at what constitutes quantum theory shows that wave mechanics is but one mathematical manifestation or representation of an underlying, more general theory whose principles can be applied to all of the above examples. The language of this more general theory is the language of vector spaces, of state vectors and of Hermitean operators and observables, of eigenvalues and eigenvectors, of linear superpositions of states, of time development operators, and so on. This language contains the essence of quantum mechanics, and it is this general version of quantum mechanics that this course is designed to introduce you to. The aim in mind is to show just how all-encompassing the theory is, and just how strange it is.

The starting point will be a quick review of the history of quantum mechanics, with the aim of summarizing the essence of the wave mechanical point of view. Following this, a study will be made of the one experiment that is supposed to embody all of the mystery of quantum mechanics – the double slit interference experiment. A closer analysis of this experiment also leads to the introduction of a new notation – the Dirac notation – along with a new interpretation in terms of vectors in a Hilbert space. Subsequently, working with this general way of presenting quantum mechanics, the physical content of the theory will be developed.