MACQUARIE UNIVERSITY | FACULTY OF SCIENCE

Quantum Physics Notes

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Foreword

THE world of our every-day experiences – the world of the not too big (compared to, say, a galaxy), and the not too small, (compared to something the size and mass of an atom), and where nothing moves too fast (compared to the speed of light) – is the world that is mostly directly accessible to our senses. This is the world usually more than adequately described by the theories of classical physics that dominated the nineteenth century: Newton's laws of motion, including his law of gravitation, Maxwell's equations for the electromagnetic field, and the three laws of thermodynamics. These classical theories are characterized by, amongst other things, the notion that there is a 'real' world out there, one that has an existence independent of ourselves, in which, for instance, objects have a definite position and momentum which we could measure to any degree of accuracy, limited only by our experimental ingenuity. According to this view, the universe is evolving in a way completely determined by these classical laws, so that if it were possible to measure the positions and momenta of all the constituent particles of the universe, and we knew all the forces that acted between the particles, then we could in principle predict to what ever degree of accuracy we desire, exactly how the universe (including ourselves) will evolve. Everything is predetermined – there is no such thing as free will, there is no room for chance. Anything apparently random only appears that way because of our ignorance of all the information that we would need to have to be able to make precise predictions.

This rather gloomy view of the nature of our world did not survive long into the twentieth century. It was the beginning of that century which saw the formulation of, not so much a new physical theory, but a new set of fundamental principles that provides a framework into which all physical theories must fit: quantum mechanics. To a greater or lesser extent all natural phenomena appear to be governed by the principles of quantum mechanics, so much so that this theory constitutes what is undoubtedly the most successful theory of modern physics. One of the crucial consequences of quantum mechanics was the realization that the world view implied by classical physics, as outlined above, was no longer tenable. Irreducible randomness was built into the laws of nature. The world is inherently probabilistic in that events can happen without a cause, a fact first stumbled on by Einstein, but never fully accepted by him. But more than that, quantum mechanics admits the possibility of an interconnectedness or an 'entanglement' between physical systems, even those possibly separated by vast distances, that has no analogue in classical physics, and which plays havoc with our strongly held presumptions that there is an objectively real world 'out there'.

Quantum mechanics is often thought of as being the physics of the very small as seen through its successes in describing the structure and properties of atoms and molecules – the chemical properties of matter – the structure of atomic nuclei and the properties of elementary particles. But this is true only insofar as the fact that peculiarly quantum effects are most readily observed at the atomic level. In the everyday world that we usually experience, where the classical laws of Newton and Maxwell seem to be able to explain so much, it quickly becomes apparent that classical theory is unable to explain many things e.g. why a solid is 'solid', or why a hot object has the colour that it does. Beyond that, quantum mechanics is needed to explain radioactivity, how semiconducting devices – the backbone of modern high technology – work, the origin of superconductivity, what makes a laser do what it does .... Even on the very large scale, quantum effects leave their mark in unexpected ways: the galaxies spread throughout the universe are believed to be macroscopic manifestations of microscopic quantum-induced inhomogeneities present shortly after the birth of the universe, when the universe itself was tinier than an atomic nucleus and almost wholly quantum mechanical. Indeed, the marriage of quantum mechanics – the physics of the very small – with general relativity – the physics of the very large – is believed by some to be the crucial step in formulating a general 'theory of everything' that will hopefully contain all the basic laws of nature in one package.

The impact of quantum mechanics on our view of the world and the natural laws that govern it, cannot be underestimated. But the subject is not entirely esoteric. Its consequences have been exploited in many ways that have an immediate impact on the quality of our lives. The economic impact of quantum mechanics cannot be ignored: it has been estimated that about 30% of the gross national product of the United States is based on inventions made possible by quantum mechanics. If anyone aims to have anything like a broad understanding of the sciences that underpin modern technology, as well as obtaining some insight into the modern view of the character of the physical world, then some knowledge and understanding of quantum mechanics is essential. In the broader community, the peculiar things that quantum mechanics says about the way the world works has meant that general interest books on quantum mechanics and related subjects continue to popular with laypersons. This is clear evidence that the community at large and not just the scientific and technological community are very interested in what quantum mechanics has to say. Note that even the term 'quantum' has entered the vernacular – it is the name of a car, a market research company, and a dishwasher amongst other things!! The phrase 'quantum jump' or 'quantum leap' is now in common usage, and incorrectly too: a quantum jump is usually understood to represent a substantial change whereas a quantum jump in its physics context is usually something that is very small.

The successes of quantum mechanics have been extraordinary. Following the principles of quantum mechanics, it is possible to provide an explanation of everything from the state of the universe immediately after the big bang, to the structure of DNA, to the colour of your socks. Yet for all of that, and in spite of the fact that the theory is now roughly 100 years old, if Planck's theory of black body radiation is taken as being the birth of quantum mechanics, it as true now as it was then that no one truly understands the theory, though in recent times, a greater awareness has developed of what quantum mechanics is all about: as well as being a physical theory, it is also a theory of information, that is, it is a theory concerning what information we can gain about the world about us – nature places limitations on what we can 'know' about the physical world, but it also gives us greater freedoms concerning what we can do with this 'quantum information' (as compared to what we could expect classically), as realized by recent developments in quantum computation, quantum teleportation, quantum cryptography and so on. For instance, hundreds of millions of dollars are being invested world-wide on research into quantum computing. Amongst other things, if quantum computing ever becomes realizable, then all security protocols used by banks, defense, and businesses can be cracked on the time scale on the order of months, or maybe a few years, a task that would take a modern classical computer 10^{10} years to achieve! On the other hand, quantum cryptography, an already functioning technology, offers us perfect security. It presents a means by which it is *always* possible to know if there is an eavesdropper listening in on what is supposed to be a secure communication channel. But even if the goal of building a quantum computer is never reached, trying to achieve it has meant an explosion in our understanding of the quantum information aspects of quantum mechanics, and which may perhaps one day finally lead us to a full understanding of quantum mechanics itself.

The Language of Quantum Mechanics

As mentioned above, quantum mechanics provides a framework into which all physical theories must fit. Thus any of the theories of physics, such as Maxwell's theory of the electromagnetic field, or Newton's description of the mechanical properties of matter, or Einstein's general relativistic theory of gravity, or any other conceivable theory, must be constructed in a way that respects the edicts of quantum mechanics. This is clearly a very general task, and as such it is clear that quantum mechanics must refer to some deeply fundamental, common feature of all these theories. This common feature is the *information* that can be known about the physical state of a physical system. Of course, the theories of classical physics are built on the information gained about the physical world, but the difference here is that quantum mechanics provides a set of rules regarding the information that can be gained about the state of *any* physical system and how this information can be processed, that are quite distinct from those implicit in classical physics. These rules tell us, amongst other things, that it is possible to have exact information about *some* physical properties of a system, but everything else is subject to the laws of probability.

To describe the quantum properties of any physical system, a new mathematical language is required as compared to that of classical mechanics. At its heart quantum mechanics is a mathematically abstract subject expressed in terms of the language of complex linear vector spaces – in other words, linear algebra. In fact, it was in this form that quantum mechanics was first worked out, by Werner Heisenberg, in the 1920s who showed how to represent the physically observable properties of systems in terms of matrices. But not long after, a second version of quantum mechanics appeared, that due to Erwin Schrödinger. Instead of being expressed in terms of matrices and vectors, it was written down in the terms of waves propagating through space and time (at least for a single particle system). These waves were represented by the so-called wave function $\Psi(x, t)$, and the equation that determined the wave function in any given circumstance was known as the Schrödinger equation.

This version of the quantum theory was, and still is, called 'wave mechanics'. It is fully equivalent to Heisenberg's version, but because it is expressed in terms of the then more familiar mathematical language of functions and wave equations, and as it was usually far easier to solve Schrödinger's equation than it was to work with (and understand) Heisenberg's version, it rapidly became 'the way' of doing quantum mechanics, and stayed that way for most of the rest of the 20th century. Its most usual application, built around the wave function Ψ and the interpretation of $|\Psi|^2$ as giving the probability of finding a particle in some region in space, is to describing the structure of matter at the atomic level where the positions of the particles is important, such as in the distribution in space of electrons and nuclei in atomic, molecular and solid state physics. But quantum mechanics is much more than the mechanics of the wave function, and its applicability goes way beyond atomic, molecular or solid state theory. Wave mechanics is but one mathematical manifestation or representation of this underlying, more general theory, and in a sense is one step removed from the mathematical language that highlights the informational interpretation of quantum mechanics. The language of this more general theory is the language of vector spaces, of state vectors and linear superpositions of states, of Hermitean operators and observables, of eigenvalues and eigenvectors, of time evolution operators, and so on. As the subject has matured in the latter decades of the 20th century and into the 21st century, and with the development of the 'quantum information' interpretation of quantum mechanics, more and more the tendency is to move away from wave mechanics to the more abstract linear algebra version, chiefly expressed in the notation due to Dirac. It is this more general view of quantum mechanics that is presented in these notes.

The starting point is a look at what distinguishes quantum mechanics from classical mechanics, followed by a quick review of the history of quantum mechanics, with the aim of summarizing the essence of the wave mechanical point of view. A study is then made of the one experiment that is supposed to embody all of the mystery of quantum mechanics – the double slit interference experiment. A closer analysis of this experiment also leads to the introduction of a new notation – the Dirac notation – along with a new interpretation in terms of vectors in a Hilbert space. Subsequently, working with this general way of presenting quantum mechanics, the physical content of the theory is developed.

The overall approach adopted here is one of inductive reasoning, that is the subject is developed by a process of trying to see what might work, or what meaning might be given to a certain mathematical or physical result or observation, and then testing the proposal against the scientific evidence. The procedure is not a totally logical one, but the result *is* a logical edifice that is only logical after the fact, i.e. the justification of what is proposed is based purely on its ability to agree with what is known about the physical world.

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