

## Chapter 4

# The Two Slit Experiment

THIS experiment is said to illustrate the essential mystery of quantum mechanics<sup>1</sup>. This mystery is embodied in the apparent ability of a system to exhibit properties which, from a classical physics point-of-view, are mutually contradictory. We have already touched on one such instance, in which the same physical system can exhibit under different circumstances, either particle or wave-like properties, otherwise known as wave-particle duality. This property of physical systems, otherwise known as ‘the superposition of states’, must be mirrored in a mathematical language in terms of which the behaviour of such systems can be described and in some sense ‘understood’. As we shall see in later chapters, the two slit experiment is a means by which we arrive at this new mathematical language.

The experiment will be considered in three forms: performed with macroscopic particles, with waves, and with electrons. The first two experiments merely show what we expect to see based on our everyday experience. It is the third which displays the counterintuitive behaviour of microscopic systems – a peculiar combination of particle and wave like behaviour which cannot be understood in terms of the concepts of classical physics. The analysis of the two slit experiment presented below is more or less taken from Volume III of the Feynman Lectures in Physics.

### 4.1 An Experiment with Bullets

Imagine an experimental setup in which a machine gun is spraying bullets at a screen in which there are two narrow openings, or slits which may or may not be covered. Bullets that pass through the openings will then strike a further screen, the detection or observation screen, behind the first, and the point of impact of the bullets on this screen are noted.

Suppose, in the first instance, that this experiment is carried out with only one slit opened, slit 1 say. A first point to note is that the bullets arrive in ‘lumps’, (assuming indestructible bullets), i.e. every bullet that leaves the gun (and makes it through the slits) arrives as a whole somewhere on the detection screen. Not surprisingly, what would be observed is the tendency for the bullets to strike the screen in a region somewhere immediately opposite the position of the open slit, but because the machine gun is firing erratically, we would expect that not all the bullets would strike the screen in exactly the same spot, but to strike the screen at random, though always in a region roughly opposite the opened slit. We can represent this experimental outcome by a curve  $P_1(x)$  which is simply such that

$$P_1(x) \delta x = \text{probability of a bullet landing in the range } (x, x + \delta x). \quad (4.1)$$

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<sup>1</sup>Another property of quantum systems, known as ‘entanglement’, sometimes vies for this honour, but entanglement relies on this ‘essential mystery’ we are considering here

If we were to cover this slit and open the other, then what we would observe is the tendency for the bullets to strike the screen opposite this opened slit, producing a curve  $P_2(x)$  similar to  $P_1(x)$ . These results are indicated in Fig. (4.1).

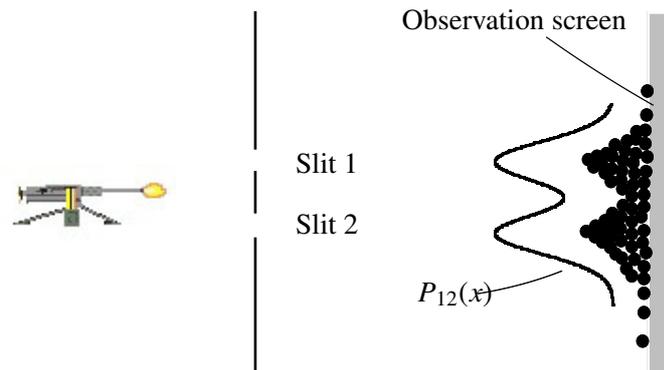


Figure 4.1: The result of firing bullets at the screen when only one slit is open. The curves  $P_1(x)$  and  $P_2(x)$  give the probability densities of a bullet passing through slit 1 or 2 respectively and striking the screen at  $x$ .

Finally, suppose that both slits are opened. We would then observe the bullets would sometimes come through slit 1 and sometimes through slit 2 – varying between the two possibilities in a random way – producing two piles behind each slit in a way that is simply the sum of the results that would be observed with one or the other slit opened, i.e.

$$P_{12}(x) = P_1(x) + P_2(x) \quad (4.2)$$

Figure 4.2: The result of firing bullets at the screen when both slits are open. The bullets accumulate on an observation screen, forming two small piles opposite each slit. The curve  $P_{12}(x)$  represents the probability density of bullets landing at point  $x$  on the observation screen.



In order to quantify this last statement, we construct a histogram with which to specify the way the bullets spread themselves across the observation screen. We start by assuming that this screen is divided up into boxes of width  $\delta x$ , and then count the number of bullets that land in each box. Suppose that the number of bullets fired from the gun is  $N$ , where  $N$  is a large number. If  $\delta N(x)$  bullets land in the box occupying the range  $x$  to  $x + \delta x$  then we can plot a histogram of  $\delta N/N\delta x$ , the fraction of all the bullets that arrive, per unit length, in each interval over the entire width of the screen.

If the number of bullets is very large, and the width  $\delta x$  sufficiently small, then the histogram will define a smooth curve,  $P(x)$  say. What this quantity  $P(x)$  represents can be gained by considering

$$P(x) \delta x = \frac{\delta N}{N} \quad (4.3)$$

which is the fraction of all the bullets fired from the gun that end up landing on the screen in region  $x$  to  $x + \delta x$ . In other words, if  $N$  is very large,  $P(x) \delta x$  approximates to the *probability* that any given bullet will arrive at the detection screen in the range  $x$  to  $x + \delta x$ . An illustrative example is

given in Fig. (4.3) of the histogram obtained when 133 bullets strike the observation screen when both slits are open. In this figure the approximate curve for  $P(x)$  also plotted, which in this case will be  $P_{12}(x)$ .

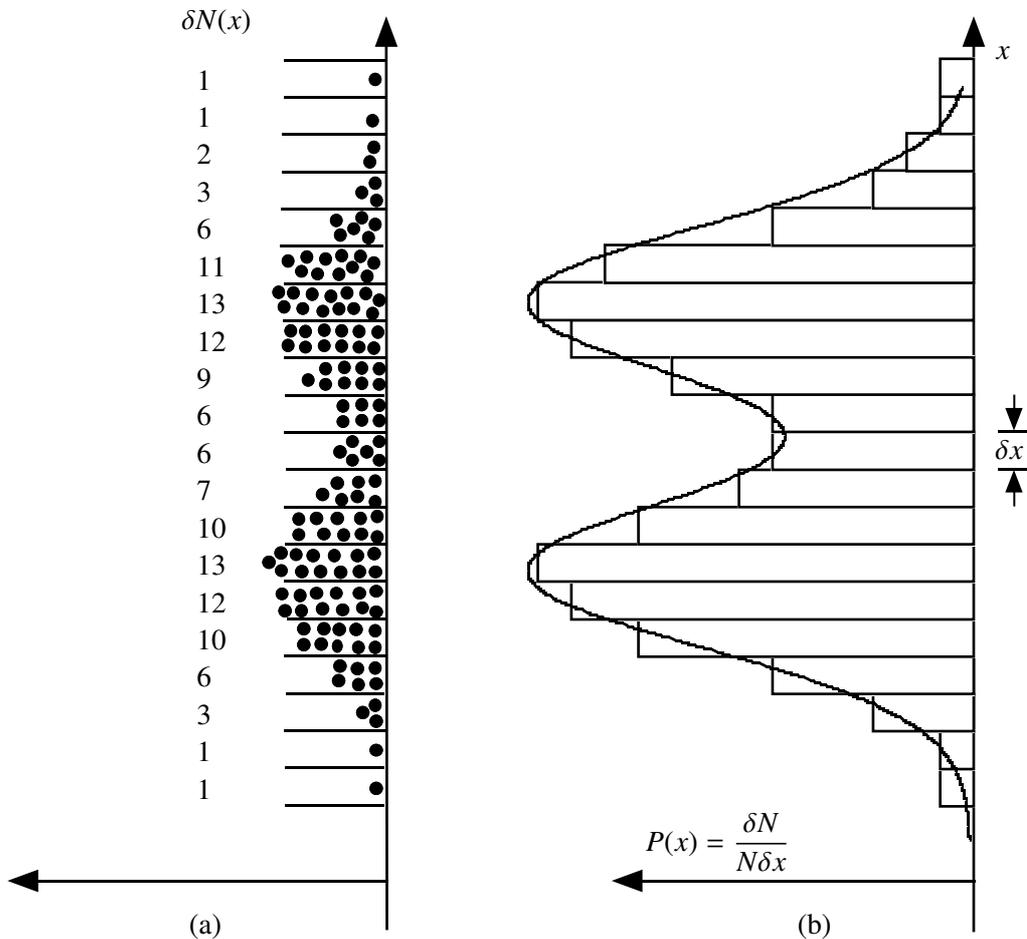


Figure 4.3: Bullets that have passed through the first screen collected in boxes all of the same size  $\delta x$ . (a) The number of bullets that land in each box is presented. There are  $\delta N(x)$  bullets in the box between  $x$  and  $x + \delta x$ . (b) A histogram is formed from the ratio  $P(x) \approx \delta N/N\delta x$  where  $N$  is the total number of bullets fired at the slits.

We can do the same in the two cases in which one or the other of the two slits are open. Thus, if slit 1 is open, then we get the curve  $P_1(x)$  in Fig. (4.2(a)), while if only slit 2 is open, we get  $P_2(x)$  such as that in Fig. (4.2(b)). What we are then saying is that if we leave both slits open, then the result will be just the sum of the two single slit curves, i.e.

$$P_{12}(x) = P_1(x) + P_2(x). \quad (4.4)$$

In other words, the *probability* of a bullet striking the screen in some region  $x$  to  $x + \delta x$  when both slits are opened is just the sum of the probabilities of the bullet landing in region when one slit and then the other is closed. This is all perfectly consistent with what we understand about the properties and behaviour of macroscopic objects — they arrive in indestructible lumps, and the probability observed with two slits open is just the sum of the probabilities with each open individually.

## 4.2 An Experiment with Waves

Now repeat the experiment with waves. For definiteness, let us suppose that the waves are light waves of wavelength  $\lambda$ . The waves pass through the slits and then impinge on the screen where we measure the intensity of the waves as a function of position along the screen.

First perform this experiment with one of the slits open, the other closed. The resultant intensity distribution is then a curve which peaks behind the position of the open slit, much like the curve obtained in the experiment using bullets. Call it  $I_1(x)$ , which we know is just the square of the amplitude of the wave incident at  $x$  which originated from slit 1. If we deal only with the electric field, and let the amplitude<sup>2</sup> of the wave at  $x$  at time  $t$  be  $E(x, t) = E(x) \exp(-i\omega t)$  in complex notation, then the intensity of the wave at  $x$  will be

$$I_1(x) = |E_1(x, t)|^2 = E_1(x)^2. \quad (4.5)$$

Close this slit and open the other. Again we get a curve which peaks behind the position of the open slit. Call it  $I_2(x)$ . These two outcomes are illustrated in Fig. (4.4).



Figure 4.4: The result of directing waves at a screen when only one slit is open. The curves  $I_1(x)$  and  $I_2(x)$  give the intensities of the waves passing through slit 1 or 2 respectively and reaching the screen at  $x$ . (They are just the central peak of a single slit diffraction pattern.)

Now open both slits. What results is a curve on the screen  $I_{12}(x)$  which oscillates between maxima and minima – an interference pattern, as illustrated in Fig. (4.5). In fact, the theory of interference of waves tells us that

$$\begin{aligned} I_{12}(x) &= |E_1(x, t) + E_2(x, t)|^2 \\ &= I_1(x) + I_2(x) + 2E_1E_2 \cos(2\pi d \sin \theta / \lambda) \\ &= I_1(x) + I_2(x) + 2\sqrt{I_1(x)I_2(x)} \cos \delta \end{aligned} \quad (4.6)$$

where  $\delta = 2\pi d \sin \theta / \lambda$  is the phase difference between the waves from the two slits arriving at point  $x$  on the screen at an angle  $\theta$  to the straight through direction. This is certainly quite different from what was obtained with bullets where there was no interference term. Moreover, the detector does not register the arrival of individual lumps of wave energy: the waves arrive everywhere at the same time, and the intensity can have any value at all.

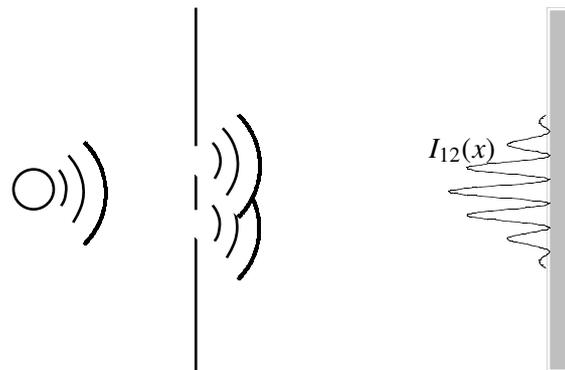


Figure 4.5: The usual two slit interference pattern.

<sup>2</sup>The word ‘amplitude’ is used here to represent the value of the wave at some point in time and space, and is *not* used to represent the maximum value of an oscillating wave.

### 4.3 An Experiment with Electrons

We now repeat the experiment for a third time, but in this case we use electrons. Here we imagine that there is a beam of electrons incident normally on a screen with the two slits, with all the electrons having the same energy  $E$  and momentum  $p$ . The screen is a fluorescent screen, so that the arrival of each electron is registered as a flash of light – the signature of the arrival of a *particle* on the screen. It might be worthwhile pointing out that the experiment to be described here was not actually performed until the very recent past, and even then not quite in the way described here. Nevertheless, the conclusions reached are what would be expected on the basis of what is now known about quantum mechanics from a multitude of other experiments. Thus, this largely hypothetical experiment (otherwise known as a thought experiment or gedanken experiment) serves to illustrate the kind of behaviour that quantum mechanics would produce, and in a way that can be used to establish the basic principles of the theory.

Let us suppose that the electron beam is made so weak that only one electron passes through the apparatus at a time. What we will observe on the screen will be individual point-flashes of light, and only one at a time as there is only one electron passing through the apparatus at a time. In other words, the electrons are arriving at the screen in the manner of particles, i.e. arriving in lumps. If we close first slit 2 and observe the result we see a localization of flashes in a region directly opposite slit 1. We can count up the number of flashes in a region of size  $\delta x$  to give the fraction of flashes that occur in the range  $x$  to  $x + \delta x$ , as in the case of the bullets. As there, we will call the result  $P_1(x)$ . Now do the same with slit 1 closed and slit 2 opened. The result is a distribution described by the curve  $P_2(x)$ . These two curves give, as in the case of the bullets, the probabilities of the electrons striking the screen when one or the other of the two slits are open. But, as in the case of the bullets, this randomness is not to be seen as all that unexpected – the electrons making their way from the source through the slits and then onto the screen would be expected to show evidence of some inconsistency in their behaviour which could be put down to, for instance, slight variations in the energy and direction of propagation of each electron as it leaves the source.

Now open both slits. What we notice now is that these flashes do not always occur at the same place — in fact they appear to occur randomly across the screen. But there is a pattern to this randomness. If the experiment is allowed to continue for a sufficiently long period of time, what is found is that there is an accumulation of flashes in some regions of the screen, and very few, or none, at other parts of the screen. Over a long enough observation time, the accumulation of detections, or flashes, forms an interference pattern, a characteristic of wave motion i.e. in contrast to what happens with bullets, we find that, for electrons,  $P_{12}(x) \neq P_1(x) + P_2(x)$ . In fact, we obtain a result of the form

$$P_{12}(x) = P_1(x) + P_2(x) + 2\sqrt{P_1(x)P_2(x)}\cos\delta \quad (4.7)$$

so we are forced to conclude that this is the result of the interference of two waves propagating from each of the slits. One feature of the waves, namely their wavelength, can be immediately determined from the separation between successive maxima of the interference pattern. It is found that  $\delta = 2\pi d \sin\theta/\lambda$  where  $\lambda = h/p$ , and where  $p$  is the momentum of the incident electrons. Thus, these waves can be identified with the de Broglie waves introduced earlier, represented by the wave function  $\Psi(x, t)$ .

So what is going on here? If electrons are particles, like bullets, then it seems clear that the electrons go *either* through slit 1 *or* through slit 2, because that is what particles would do. The behaviour of the electrons going through slit 1 should then not be affected by whether slit 2 is opened or closed as those electrons would go nowhere near slit 2. In other words, we have to expect that  $P_{12}(x) = P_1(x) + P_2(x)$ , but this not what is observed. It appears that we must abandon the idea that the particles go through one slit or the other. But if we want to retain the mental picture of electrons as particles, we must conclude that the electrons pass through *both* slits in some way,

because it is only by ‘going through both slits’ that there is any chance of an interference pattern forming. After all, the interference term depends on  $d$ , the separation between the slits, so we must expect that the particles must ‘know’ how far apart the slits are in order for the positions that they strike the screen to depend on  $d$ , and they cannot ‘know’ this if each electron goes through only one slit. We could imagine that the electrons determine the separation between slits by supposing that they split up in some way, but then they will have to subsequently recombine before striking the screen since all that is observed is single flashes of light. So what comes to mind is the idea of the electrons executing complicated paths that, perhaps, involve them looping back through each slit, which is scarcely believable. The question would have to be asked as to why the electrons execute such strange behaviour when there are a pair of slits present, but do not seem to when they are moving in free space. There is no way of understanding the double slit behaviour in terms of a particle picture only.

### 4.3.1 Monitoring the slits: the Feynman microscope

We may argue that one way of resolving the issue is to actually monitor the slits, and look to see when an electron passes through each slit. There are many many ways of doing this. One possibility, a variation on the Heisenberg microscope discussed in Section 3.3.1, was proposed by Feynman, and is known as the Feynman light microscope. The experiment consists of shining a light on the slits so that, if an electron goes through a slit, then it scatters some of this light, which can then be observed with a microscope. We immediately know what slit the electron passed through. Remarkably, as a consequence of gaining this knowledge, what is found is that the interference pattern disappears, and what is seen on the screen is the same result as for bullets.

The essence of the argument is exactly that presented in Section 3.3.1 in the discussion of the Heisenberg microscope experiment. The aim is to resolve the position of the electron to an accuracy of at least  $\delta x \sim d/2$ , but as a consequence, we find that the electron is given a random kick in momentum of magnitude  $\delta p \sim 2h/d$ , (see Eq. (3.17)).

Thus, an electron passing through, say, the upper slit, could be deflected by an amount up to an angle of  $\delta\theta$  where

$$\delta\theta \sim \delta p/p \quad (4.8)$$

where  $p = h/\lambda$  is the momentum of the electron and  $\lambda$  its de Broglie wavelength. Thus we find

$$\delta\theta \sim \frac{2h/d}{h/\lambda} = 2\lambda/d \quad (4.9)$$

i.e. the electron will be deflected through this angle, while at the same time, the photon will be seen as coming from the position of the upper slit.

Since the angular separation between a maximum of the interference pattern and a neighbouring minimum is  $\lambda/2d$ , the uncertainty in the deflection of an observed electron is at least enough to displace an electron from its path heading for a maximum of the interference pattern, to one where it will strike the screen at the position of a minimum. The overall result is to wipe out the interference pattern. But we could ask why it is that the photons could be scattered through a range of

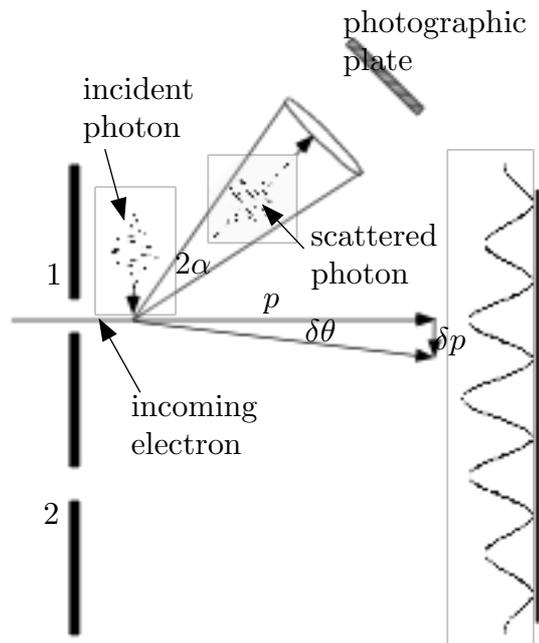


Figure 4.6: An electron with momentum  $p$  passing through slit 1 scatters a photon which is observed through a microscope. The electron gains momentum  $\delta p$  and is deflected from its original course.

angles, i.e. why do we say ‘up to an angle of  $\delta\theta$ ’? After all, the experiment could be set up so that every photon sent towards the slits has the same momentum every time, and similarly for every electron, so every scattering event should produce the same result, and the outcome would merely be the interference pattern displaced sideways. It is here where quantum randomness comes in. The amount by which the electron is deflected depends on through what angle the photon has been scattered, which amounts to knowing what part of the observation lens the photon goes through on its way to the photographic plate. But we cannot know this unless we ‘look’ to see where in the lens the photon passes. So there is an uncertainty in the position of the photon as it passes through the lens which, by the uncertainty relation, implies an uncertainty in the momentum of the photon in a direction parallel to the lens, so we cannot know with precision where the photon will land on the photographic plate. As a consequence, the position that each scattered photon will strike the photographic plate will vary randomly from one scattered photon to the next in much the same way as the position that the electron strikes the observation screen varies in a random fashion in the original (unmonitored) two slit experiment. The uncertainty as to which slit the electron has passed through has been transferred to the uncertainty about which part of the lens the scattered photon passes through.

So it would seem that by trying to determine through which slit the electron passes, the motion of the electron is always sufficiently disturbed so that no interference pattern is observed. As far as it is known, *any* experiment that can be devised – either a real experiment or a gedanken (i.e. a thought) experiment – that involves directly observing through which slit the electrons pass always results in a disturbance in the motion of the electron sufficient for the interference pattern to be wiped out. An ‘explanation’ of why this occurs, of which the above discussion is an example, could perhaps be constructed in each case, but in any such explanation, the details of the way the physics conspires to produce this result will differ from one experiment to the other<sup>3</sup>. These explanations often require a mixture of classical and quantum concepts, i.e. they have a foot in both camps and as such are not entirely satisfactory, but more thorough studies of, for instance, the Feynman microscope experiment using a fully quantum mechanical approach, confirm the results discussed here. So it may come as a surprise to find that there need not be any direct physical interaction between the electrons and the apparatus used to monitor the slits, so there is no place for the kind of quantum/classical explanation given above. As long as there is *information* available regarding which slit the electron passes through, there is no interference, as the following example illustrates.

### 4.3.2 The Role of Information: The Quantum Eraser

Suppose we construct the experimental set-up illustrated in Fig. (4.7). The actual experiment is performed with photons, but we will stick with an ‘electron version’ here. In this experiment, the electron source sends out *pairs* of particles. One of the pair (sometimes called the ‘signal’ particle in the photon version of the experiment) heads towards the slits, while the other (the ‘idler’) heads off at right angles. What we find here is that, if we send out many electrons in this fashion, then *no interference pattern* is built up on the observation screen, i.e. we get the result we expect if we are able to determine through which slit each electron passes. But this outcome is not as puzzling as it first might seem since, in fact, we actually do have this information: for each emitted electron there is a correlated idler particle whose direction of motion will tell us in what direction the electron is headed. Thus, in principle, we can determine which slit an electron will pass through by choosing

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<sup>3</sup>Einstein, who did not believe quantum mechanics was a complete theory, played the game of trying to find an experimental setup that would bypass the uncertainty relation i.e. to know which slit an electron passes through AND to observe an interference pattern. Bohr answered all of Einstein’s challenges, including, in one instance, using Einstein’s own theory of general relativity to defeat a proposal of Einstein’s concerning another form of the uncertainty relation, the time-energy uncertainty relation. It was at this point that Einstein abandoned the game, but not his attitude to quantum mechanics.

to *detect* the correlated idler particle by turning on detectors *a* and *b*. Thus, if we detect a particle at *a*, then electron is heading for slit 1.

However, we do not actually have to carry out this detection. The sheer fact that this information is available for us to access if we choose to do so is enough for the interference pattern not to appear. However, if there is only one detector capable of detecting either of the idler particles, then we cannot tell, if this detector registers a count, which idler particle it detected. In this case, the ‘which path’ information has been *erased*. If we erase this information for every electron emitted, there will appear an interference pattern even though *there has been no physical interaction with the electrons passing through the slits!!* This example suggests that provided there is information present as to which slit an electron passes through, even if we do not access this information, then there is no interference pattern. This is an example of quantum entanglement: the direction of the electron heading towards the slits is *entangled* with the direction of the ‘information carrier’ idler particle.

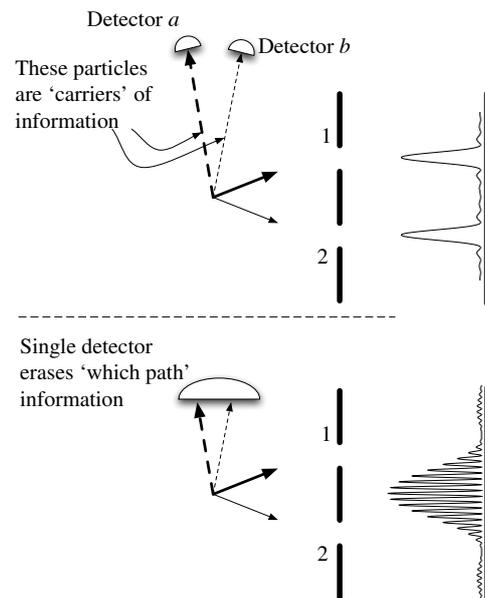


Figure 4.7: Illustrating the ‘quantum eraser’ experiment. In the first set-up, the information on ‘which slit’ is encoded in the auxiliary particle, and there is no interference. In the second, this information is erased and interference is observed.

This more general analysis shows that the common feature in all these cases is that the electron becomes ‘entangled’ with some other system, in such a manner that this other system carries with it information about the electron. In this example, entanglement is present right from the start of the experiment, while in the monitoring-by-microscope example, entanglement with the scattered photon has been actively established, with information on the position of the electron encoded in the photon. But that is all that we require, i.e. that the information be so encoded – we do not need to have some one consciously observe these photons for the interference pattern to disappear, it is merely sufficient that this information be available. It is up to use to choose to access it or not.

What is learned from this is that if information is in principle available on the position of the electron or, in other words, if we want to be anthropomorphic about all this, if we can in principle *know* which slit the electron goes through by availing ourselves of this information, even without interfering with the electron, then the observed pattern of the screen is the same as found with bullets: no interference, with  $P_{12}(x) = P_1(x) + P_2(x)$ . If this information is not available at all, then we are left with the uncomfortable thought that in some sense each electron passes through both slits, resulting in the formation of an interference pattern, the signature of wave motion. Thus, the electrons behave either like particles or like waves, depending on what information is extant, a dichotomy that is known as wave-particle duality.

### 4.3.3 Wave-particle duality

So it appears that if we gain information on which slit the electron passes through then we lose the interference pattern. Gaining information on which slit means we are identifying the electron as a particle — only particles go through one slit or the other. But once this identification is made, say an electron has been identified as passing through slit 1, then that electron behaves thereafter

like a particle that has passed through one slit only, and so will contribute to the pattern normally produced when only slit 1 is open. And similarly for an electron identified as passing through slit 2. Thus, overall, no interference pattern is produced. If we do not determine which slit, the electrons behave as waves which are free to probe the presence of each slit, and so will give rise to an interference pattern.

This dichotomy, in which the same physical entity, an electron (or indeed, any particle), can be determined to be *either* a particle *or* a wave depending on what experiment is done to observe its properties, is known as wave-particle duality.

But in all instances, the inability to observe both the wave and particle properties – i.e. to know which slit a particle passes through AND for there to be an interference pattern – can be traced back to the essential role played by the uncertainty relation  $\Delta x \Delta p \geq \frac{1}{2} \hbar$ . As Feynman puts it, this relation ‘protects’ quantum mechanics. But note that this is NOT the measurement form of the uncertainty relation: there is no need for a disturbance to act on the particle as a consequence of a measurement, for quantum mechanics to be protected, as we have just seen.

The fact that *any* situation for which there is information on which slit the electron passes through always results in the non-appearance of the interference pattern suggests that there is a deep physical principle at play here – a law or laws of nature – that overrides, irrespective of the physical set-up, any attempt to have information on where a particle is, and to observe interference effects. The laws are the laws of quantum mechanics, and one of their consequences is the uncertainty principle, that is  $\Delta x \Delta p \geq \frac{1}{2} \hbar$ . If we want to pin down the slit through which the particle has passed with, say, 97% certainty, then the consequent uncertainty  $\Delta x$  in the position of the particle can be estimated to be  $\Delta x \approx 0.17d$ , where  $d$  is the separation of the slits<sup>4</sup>. But doing so implies that there is an uncertainty in the sideways momentum of the electron given by  $\Delta p \gtrsim \hbar/0.17d$ . This amounts to a change in direction through an angle

$$\Delta\theta = \frac{\Delta p}{p} \gtrsim \frac{\lambda}{2d} \quad (4.10)$$

Since the angular separation between a minimum and a neighbouring maximum of the diffraction pattern is  $\lambda/2d$ , it is clear that the uncertainty in the sideways momentum arising from trying to observe through which slit the particle passes is enough to displace a particle from a maximum into a neighbouring minimum, washing out the interference pattern.

Using the uncertainty principle in this way does not require us to invoke any ‘physical mechanism’ to explain why the interference pattern washes out; only the abstract requirements of the uncertainty principle are used. Nothing is said about how the position of the particle is pinned down to within an uncertainty  $\Delta x$ . If this information is provided, then a physical argument, of sorts, as in the case of Heisenberg’s microscope, that mixes classical and quantum mechanical ideas might be possible that explains why the interference pattern disappears. However, the details of the way the physics conspires to produce this result will differ from one experiment to the other. And in any case, there might not be any physical interaction at all, as we have just seen. It is the laws of quantum mechanics (from which the uncertainty principle follows) that tell us that the interference

<sup>4</sup>This estimate is arrived at by assigning a probability  $P$  that the particle is at the slit with position  $d/2$  and a probability  $1 - P$  that it is at the position of the slit at  $-d/2$  based on the observed outcome of the measurement, i.e. the measurement is not perfect. The mean position of the electron is now

$$\langle x \rangle = Pd/2 - (1 - P)d/2 = (P - \frac{1}{2})d$$

and the standard deviation of this outcome is

$$(\Delta x)^2 = P(d/2 - \langle x \rangle)^2 + (1 - P)(-d/2 - \langle x \rangle)^2 = P(1 - P)d$$

so  $\Delta x = \sqrt{P(1 - P)}d$ .

pattern must disappear if we measure particle properties of the electrons, and this is so *irrespective of the particular kind of physics involved in the measurement* – the individual physical effects that may be present in one experiment or another are subservient to the laws of quantum mechanics.

#### 4.4 Probability Amplitudes

First, a summary of what has been seen so far. In the case of waves, we have seen that the total amplitude of the waves incident on the screen at the point  $x$  is given by

$$E(x, t) = E_1(x, t) + E_2(x, t) \quad (4.11)$$

where  $E_1(x, t)$  and  $E_2(x, t)$  are the waves arriving at the point  $x$  from slits 1 and 2 respectively. The intensity of the resultant interference pattern is then given by

$$\begin{aligned} I_{12}(x) &= |E(x, t)|^2 \\ &= |E_1(x, t) + E_2(x, t)|^2 \\ &= I_1(x) + I_2(x) + 2E_1E_2 \cos\left(\frac{2\pi d \sin \theta}{\lambda}\right) \\ &= I_1(x) + I_2(x) + 2\sqrt{I_1(x)I_2(x)} \cos \delta \end{aligned} \quad (4.12)$$

where  $\delta = 2\pi d \sin \theta / \lambda$  is the phase difference between the waves arriving at the point  $x$  from slits 1 and 2 respectively, at an angle  $\theta$  to the straight through direction. The point was then made that the probability density for an electron to arrive at the observation screen at point  $x$  had the same form, i.e. it was given by the same mathematical expression

$$P_{12}(x) = P_1(x) + P_2(x) + 2\sqrt{P_1(x)P_2(x)} \cos \delta. \quad (4.13)$$

so we were forced to conclude that this is the result of the interference of two waves propagating from each of the slits. Moreover, the wavelength of these waves was found to be given by  $\lambda = h/p$ , where  $p$  is the momentum of the incident electrons so that these waves can be identified with the de Broglie waves introduced earlier, represented by the wave function  $\Psi(x, t)$ . Thus, we are proposing that incident on the observation screen is the de Broglie wave associated with each electron whose total amplitude at point  $x$  is given by

$$\Psi(x, t) = \Psi_1(x, t) + \Psi_2(x, t) \quad (4.14)$$

where  $\Psi_1(x, t)$  and  $\Psi_2(x, t)$  are the amplitudes at  $x$  of the waves emanating from slits 1 and 2 respectively. Further, since  $P_{12}(x)\delta x$  is the probability of an electron being detected in the region  $x, x + \delta x$ , we are proposing that

$$|\Psi(x, t)|^2 \delta x \propto \text{probability of observing an electron in } x, x + \delta x \quad (4.15)$$

so that we can interpret  $\Psi(x, t)$  as a *probability amplitude*. This is the famous probability interpretation of the wave function first proposed by Born on the basis of his own observations of the outcomes of scattering experiments, as well as awareness of Einstein's own inclinations along these lines. Somewhat later, after proposing his uncertainty relation, Heisenberg made a similar proposal.

There are two other important features of this result that are worth taking note of:

- If the detection event can arise in two different ways (i.e. electron detected after having passed through either slit 1 or 2) and the two possibilities remain unobserved, then the total probability of detection is

$$P = |\Psi_1 + \Psi_2|^2 \quad (4.16)$$

i.e. we add the amplitudes and then square the result.

- If the experiment contains a part that even *in principle* can yield information on which of the alternate paths were followed, then

$$P = P_1 + P_2 \quad (4.17)$$

i.e. we add the probabilities associated with each path.

What this last point is saying, for example in the context of the two slit experiment, is that, as part of the experimental set-up, there is equipment that is monitoring through which slit the particle goes. Even if this equipment is automated, and simply records the result, say in some computer memory, and we do not even bother to look the results, the fact that they are still available means that we should add the probabilities. This last point can be understood if we view the process of observation of which path as introducing randomness in such a manner that the interference effects embodied in the  $\cos \delta$  are smeared out. In other words, the  $\cos \delta$  factor – which can range between plus and minus one – will average out to zero, leaving behind the sum of probability terms.

## 4.5 The Fundamental Nature of Quantum Probability

The fact that the results of the experiment performed with electrons yields outcomes which appear to vary in a random way from experiment to experiment at first appears to be identical to the sort of randomness that occurs in the experiment performed with the machine gun. In the latter case, the random behaviour can be explained by the fact that the machine gun is not a very well constructed device: it sprays bullets all over the place. This seems to suggest that simply by refining the equipment, the randomness can be reduced, in principle removing it all together if we are clever enough. At least, that is what classical physics would lead us to believe. Classical physics permits unlimited accuracy in the fixing the values of physical or dynamical quantities, and our failure to live up to this is simply a fault of inadequacies in our experimental technique.

However, the kind of randomness found in the case of the experiment performed with electrons is of a different kind. It is intrinsic to the physical system itself. We are unable to refine the experiment in such a way that we can know precisely what is going on. Any attempt to do so gives rise to unpredictable changes, via the uncertainty principle. Put another way, it is found that experiments on atomic scale systems (and possibly at macroscopic scales as well) performed under identical conditions, where everything is as precisely determined as possible, will always, in general, yield results that vary in a random way from one run of the experiment to the next. This randomness is irreducible, an intrinsic part of the physical nature of the universe.

Attempts to remove this randomness by proposing the existence of so-called ‘classical hidden variables’ have been made in the past. These variables are supposed to be classical in nature – we are simply unable to determine their values, or control them in any way, and hence give rise to the apparent random behaviour of physical systems. Experiments have been performed that test this idea, in which a certain inequality known as the Bell inequality, was tested. If these classical hidden variables did in fact exist, then the inequality would be satisfied. A number of experiments have yielded results that are clearly inconsistent with the inequality, so we are faced with having to accept that the physics of the natural world is intrinsically random at a fundamental level, and in a way that is *not* explainable classically, and that physical theories can do no more than predict the probabilities of the outcome of any measurement.