Chapter 1

Introduction

There are three fundamental theories on which modern physics is built: the theory of relativity, statistical mechanics/thermodynamics, and quantum mechanics. Each one has forced upon us the need to consider the possibility that the character of the physical world, as we perceive it and understand it on a day to day basis, may be far different from what we take for granted.

Already, the theory of special relativity, through the mere fact that nothing can ever be observed to travel faster than the speed of light, has forced us to reconsider the nature of space and time – that there is no absolute space, nor is time 'like a uniformly flowing river'. The concept of 'now' or 'the present' is not absolute, something that everyone can agree on – each person has their own private 'now'. The theory of general relativity then tells us that space and time are curved, that the universe ought to be expanding from an initial singularity (the big bang), and will possibly continue expanding until the sky, everywhere, is uniformly cold and dark.

Statistical mechanics/thermodynamics gives us the concept of entropy and the second law: the entropy of a closed system can never decrease. First introduced in thermodynamics – the study of matter in bulk, and in equilibrium – it is an aid, amongst other things, in understanding the 'direction in time' in which natural processes happen. We remember the past, not the future, even though the laws of physics do not make a distinction between the two temporal directions 'into the past' and 'into the future'. All physical processes have what we perceive as the 'right' way for them to occur – if we see something happening 'the wrong way round' it looks very odd indeed: eggs are often observed to break, but never seen to reassemble themselves. The sense of uni-directionality of events defines for us an 'arrow of time'. But what is entropy? Statistical mechanics – which attempts to explain the properties of matter in bulk in terms of the aggregate behaviour of the vast numbers of atoms that make up matter – stepped in and told us that this quantity, entropy, is not a substance in any sense. Rather, it is a measure of the degree of disorder that a physical system can possess, and that the natural direction in which systems evolve is in the direction such that, overall, entropy never decreases. Amongst other things, this appears to have the consequence that the universe, as it ages, could evolve into a state of maximum disorder in which the universe is a cold, uniform, amorphous blob – the so-called heat death of the universe.

So what does quantum mechanics do for us? What treasured view of the world is turned upside down by the edicts of this theory? It appears that quantum mechanics delivers to us a world view in which

- There is a loss of certainty – unavoidable, unremovable randomness pervades the physical world. Einstein was very dissatisfied with this, as expressed in his well-known statement: "God does not play dice with the universe." It even appears that the very process of making an observation can affect the subject of this observation in an uncontrollably random way (even if no physical contact is made with the object under observation!).

- Physical systems appear to behave as if they are doing a number of mutually exclusive things simultaneously. For instance an electron fired at a wall with two holes in it can appear to behave as if it goes through both holes simultaneously.
- Widely separated physical systems can behave as if they are entangled by what Einstein termed some 'spooky action at a distance' so that they are correlated in ways that appear to defy either the laws of probability or the rules of special relativity.

It is this last property of quantum mechanics that leads us to the conclusion that there are some aspects of the physical world that cannot be said to be objectively 'real'. For instance, in the game known as *The Shell Game*, a pea is hidden under one of three cups, which have been shuffled around by the purveyor of the game, so that bystanders lose track of which cup the pea is under. Now suppose you are a bystander, and you asked are to guess which cup the pea is under. You might be lucky and guess which cup first time round, but you might have to have another attempt to find the cup under which the pea is hidden. But whatever happens, when you do find the pea, you implicitly believe that the pea was under that cup all along. But is it possible that the pea *really* wasn't at any one of the possible positions at all, and the sheer process of looking to see which cup the pea is under, which amounts to measuring the position of the pea, 'forces' it to be in the position where it is ultimately observed to be? Was the pea 'really' there beforehand? Quantum mechanics says that, just maybe, it wasn't there all along! Einstein had a comment or two about this as well. He once asked a fellow physicist (Pascual Jordan): "Do you believe the moon exists only when you look at it?"

The above three points are all clearly in defiance of our classical view of the world, based on the theories of classical physics, which goes had-in-hand with a particular view of the world sometimes referred to as objective realism.

1.1 Classical Physics

Before we look at what quantum mechanics has to say about how we are to understand the natural world, it is useful to have a look at what the classical physics perspective is on this. According to classical physics, by which we mean pre-quantum physics, it is essentially taken for granted that there is an 'objectively real world' out there, one whose properties, and whose very existence, is totally indifferent to whether or not we exist. These ideas of classical physics are not tied to any one person – it appears to be the world-view of Galileo, Newton, Laplace, Einstein and many other scientists and thinkers – and in all likelihood reflects an intuitive understanding of reality, at least in the Western world. This view of classical physics can be referred to as 'objective reality'.

The equations of the theories of classical physics, which include Newtonian mechanics, Maxwell's theory of the electromagnetic field and Einstein's theory of general relativity, are then presumed to describe what is 'really happening' with a physical system. For example, it is assumed that every particle has a definite position and velocity and that the solution to Newton's equations for a particle in motion is a perfect representation of what the particle is 'actually doing'.

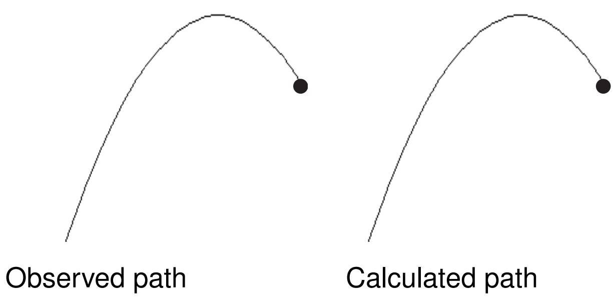


Figure 1.1: Comparison of observed and calculated paths of a tennis ball according to classical physics

Within this view of reality, we can speak about a particle moving through space, such as a tennis ball flying through the air, as if it has, at any time, a definite position and velocity. Moreover, it would have that definite position and velocity whether or not there was anyone or anything monitoring its behaviour. After all, these are properties of the tennis ball, not something attributable to our measurement efforts. Well, that is the classical way of looking at things. It is then up to us to decide whether or not we want to measure this pre-existing position and velocity. They both have definite values at any instant in time, but it is totally a function of our experimental ingenuity whether or not we can measure these values, and the level of precision to which we can measure them. There is an implicit belief that by refining our experiments — e.g. by measuring to to the 100th decimal place, then the 1000th, then the 10000th — we are getting closer and closer to the values of the position and velocity that the particle 'really' has. There is no law of physics, at least according to classical physics, that says that we definitely cannot determine these values to as many decimal places as we desire – the only limitation is, once again, our experimental ingenuity. We can also, in principle, calculate, with unlimited accuracy, the future behaviour of any physical system by solving Newton's equations, Maxwell's equations and so on. In practice, there are limits to accuracy of measurement and/or calculation, but in principle there are no such limits.

1.1.1 Classical Randomness and Ignorance of Information

Of course, we recognize, for a macroscopic object, that we cannot hope to measure all the positions and velocities of all the particles making such an object. In the instance of a litre of air in a bottle at room temperature, there are something like 10^{26} particles whizzing around in the bottle, colliding with one another and with the walls of the bottle. There is no way of ever being able to measure the position and velocities of each one of these gas particles at some instant in time. But that does not stop us from believing that each particle does in fact possess a definite position and velocity at each instant. It is just too difficult to get at the information.

Likewise, we are unable to predict the motion of a pollen grain suspended in a liquid: Brownian motion (random walk) of pollen grain due to collisions with molecules of liquid. According to classical physics, the information is 'really there' – we just can't get at it.

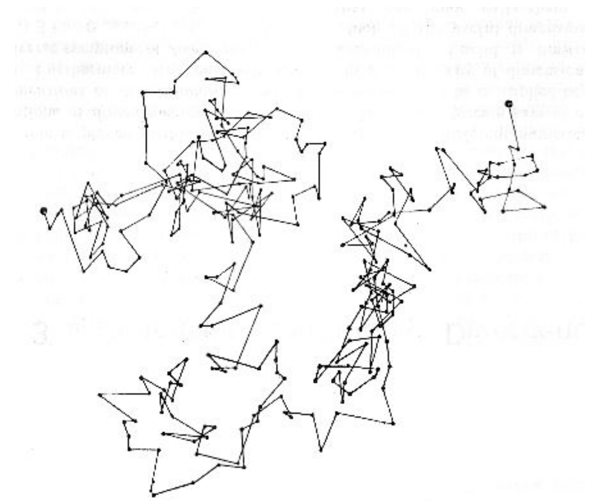


Figure 1.2: Random walk of a pollen grain suspended in a liquid.

Random behaviour only *appears* random because we do not have enough information to describe it exactly. It is not really random because we believe that if we could repeat an experiment under *exactly* identical conditions we ought to get the same result every time, and hence the outcome of the experiment would be perfectly predictable.

In the end, we accept a certain level of ignorance about the possible information that we could, in principle, have about the gas. Because of this, we cannot hope to make accurate predictions about what the future behaviour of the gas is going to be. We compensate for this ignorance by using statistical methods to work out the chances of the gas particles behaving in various possible ways. For instance, it is possible to show that the chances of all the gas particles spontaneously rushing to one end of the bottle is something like 1 in $10^{10^{26}}$ – appallingly unlikely.

The use of statistical methods to deal with a situation involving ignorance of complete information is reminiscent of what a punter betting on a horse race has to do. In the absence of complete information about each of the horses in the race, the state of mind of the jockeys, the state of the track, what the weather is going to do in the next half hour and any of a myriad other possible influences on the outcome of the race, the best that any punter can do is assign odds on each horse winning according to what information is at hand, and bet accordingly. If, on the other hand, the punter knew *everything* beforehand, the outcome of the race is totally foreordained in the mind of the punter, so (s)he could make a bet that was guaranteed to win.

According to classical physics, the situation is the same when it comes to, for instance, the evolution of the whole universe. If we knew at some instant all the positions and all the velocities of all the particles making up the universe, and all the forces that can act between these particles, then we ought to be able to calculate the entire future history of the universe. Even if we cannot carry out such a calculation, the sheer fact that it could be done tells us that the future of the universe is already ordained. This prospect was first proposed by the mathematical physicist Pierre-Simon Laplace (1749-1827) and is hence known as Laplacian determinism, and in some sense represents the classical view of the world taken to its most extreme limits. So there is no such thing, in classical physics, as true randomness. Any uncertainty we experience is purely a consequence of our ignorance – things only appear random because we do not have enough information to make precise predictions. Nevertheless, behind the scenes, everything is evolving in an entirely preordained way – everything is deterministic, there is no such thing as making a decision, free will is merely an illusion!!!

1.2 Quantum Physics

The classical world-view works fine at the everyday (macroscopic) level – much of modern engineering relies on this – but there are things at the macroscopic level that cannot be understood using classical physics, these including the colour of a heated object, the existence of solid objects .... So where does classical physics come unstuck?

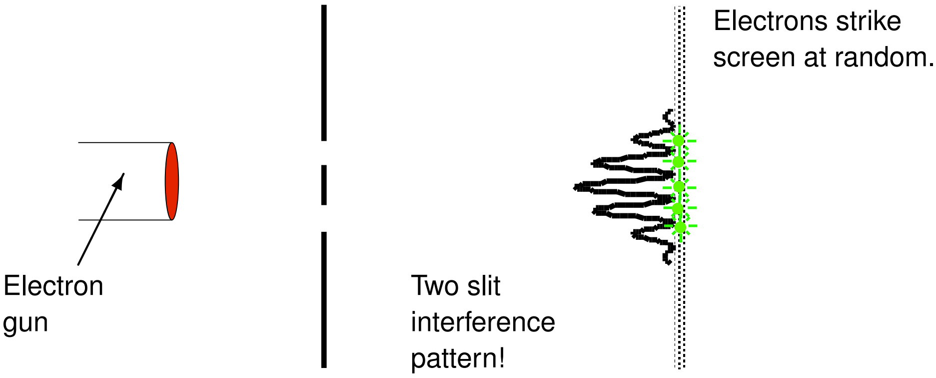
Non-classical behaviour is most readily observed for microscopic systems – atoms and molecules, but is in fact present at all scales. The sort of behaviour exhibited by microscopic systems that are indicators of a failure of classical physics are

- Intrinsic Randomness
- Interference phenomena (e.g. particles acting like waves)
- Entanglement

Intrinsic Randomness It is impossible to prepare *any* physical system in such a way that *all* its physical attributes are precisely specified at the same time – e.g. we cannot pin down both the position *and* the momentum of a particle at the same time. If we trap a particle in a tiny box, thereby giving us a precise idea of its position, and then measure its velocity, we find, after many repetitions of the experiment, that the velocity of the particle always varies in a random fashion from one measurement to the next. For instance, for an electron trapped in a box 1 micron in size, the velocity of the electron can be measured to vary by at least $\pm 50\ \mathrm{ms}^{-1}$. Refinement of the experiment *cannot* result in this randomness being reduced — it can never be removed, and making the box even tinier just makes the situation worse. More generally, it is found that for *any* experiment repeated under exactly identical conditions there will always be some physical quantity, some physical property of the systems making up the experiment, which, when measured, will always yield randomly varying results from one run of the experiment to the next. This is *not* because we do a lousy job when setting up the experiment or carrying out the measurement. The randomness is *irreducible*: it *cannot* be totally removed by improvement in experimental technique.

What this is essentially telling us is that nature places limits on how much information we can gather about any physical system. We apparently cannot know with precision as much about a system as we thought we could according to classical physics. This tempts us to ask if this missing information is still there, but merely inaccessible to us for some reason. For instance, does a particle whose position is known also have a precise momentum (or velocity), but we simply cannot measure its value? It appears that in fact this information is not missing – it is not there in the first place. Thus the randomness that is seen to occur is not a reflection of our ignorance of some information. It is not randomness that can be resolved and made deterministic by digging deeper to get at missing information – it is apparently 'uncaused' random behaviour.

Interference Microscopic physical systems can behave as if they are doing mutually exclusive things at the same time. The best known example of this is the famous two slit experiment in which electrons are fired, one at a time, at a screen in which there are two narrow slits. The electrons are observed to strike an observation screen placed beyond the screen with the slits. What is expected is that the electrons will strike this second screen in regions immediately opposite the two slits. What is observed is that the electrons arriving at this observation screen tend to arrive in preferred locations that are found to have all the characteristics of a wave-like interference pattern, i.e. the pattern formed as would be observed if it were waves (e.g. light waves) being directed towards the slits.



The detailed nature of the interference pattern is determined by the separation of the slits: increasing this separation produces a finer interference pattern. This seems to suggest that an electron, which, being a particle, can only go through one slit or the other, somehow has 'knowledge' of the position of the other slit. If it did not have that information, then it is hard to see how the electron could arrive on the observation screen in such a manner as to produce a pattern whose features are directly determined by the slit separation! And yet, if the slit through which each electron passes is observed in some fashion, the interference pattern disappears – the electrons strike the screen at positions directly opposite the slits! The uncomfortable conclusion that is forced on us is that if the path of the electron is not observed then, in some sense, it passes through both slits much as waves do, and ultimately falls on the observation screen in such a way as to produce an interference pattern, once again, much as waves do.

This propensity for quantum system to behave as if they can be two places at once, or more generally in different states at the same time, is termed 'the superposition of states' and is a singular property of quantum systems that leads to the formulation of a mathematical description based on the ideas of vector spaces.

Entanglement Suppose for reasons known only to yourself that while sitting in a hotel room in Sydney looking at a pair of shoes that you really regret buying, you decided to send one of the pair to a friend in Brisbane, and the other to a friend in Melbourne, without observing which shoe went where. It would not come as a surprise to hear that if the friend in Melbourne discovered that the shoe they received was a left shoe, then the shoe that made it to Brisbane was a right shoe, and vice versa. If this strange habit of splitting up perfectly good pairs of shoes and sending one at random to Brisbane and the other to Melbourne were repeated many times, then while it is not possible to predict for sure what the friend in, say Brisbane, will observe on receipt of a shoe, it is nevertheless always the case that the results observed in Brisbane and Melbourne were always perfectly correlated – a left shoe paired off with a right shoe.

Similar experiments can be undertaken with atomic particles, though it is the spins of pairs of particles that are paired off: each is spinning in exactly the opposite fashion to the other, so that the total angular momentum is zero. Measurements are then made of the spin of each particle when it arrives in Brisbane, or in Melbourne. Here it is not so simple as measuring whether or not the spins are equal and opposite, i.e. it goes beyond the simple example of left or right shoe, but the idea is nevertheless to measure the correlations between the spins of the particles. As was shown by John Bell, it is possible for the spinning particles to be prepared in states for which the correlation between these measured spin values is *greater* than what classical physics permits. The systems are in an 'entangled state', a quantum state that has no classical analogue. This is a conclusion that is experimentally testable via Bell's inequalities, and has been overwhelmingly confirmed. Amongst other things it seems to suggest the two systems are 'communicating' instantaneously, i.e. faster than the speed of light which is inconsistent with Einstein's theory of relativity. As it turns out, it can be shown that there is no faster-than-light communication at play here. But it can be argued that this result forces us to the conclusion that physical systems acquire some (maybe all?) properties only through the act of observation, e.g. a particle does not 'really' have a specific position until it is measured.

The sorts of quantum mechanical behaviour seen in the three instances discussed above are believed to be common to all physical systems. So what is quantum mechanics? It is saying something about *all* physical systems. Quantum mechanics is not a physical theory specific to a limited range of physical systems i.e. it is not a theory that applies only to atoms and molecules and the like. It is a meta-theory. At its heart, quantum mechanics is a set of fundamental principles that *constrain the form of physical theories themselves*, whether it be a theory describing the mechanical properties of matter as given by Newton's laws of motion, or describing the properties of the electromagnetic field, as contained in Maxwell's equations or any other conceivable theory. Another example of a meta-theory is relativity — both special and general — which places strict conditions on the properties of space and time. In other words, space and time must be treated in all (fundamental) physical theories in a way that is consistent with the edicts of relativity.

To what aspect of all physical theories do the principles of quantum mechanics apply? The principles must apply to theories as diverse as Newton's Laws describing the mechanical properties of matter, Maxwell's equations describing the electromagnetic field, the laws of thermodynamics – what is the common feature? The answer lies in noting how a theory in physics is formulated.

1.3 Observation, Information and the Theories of Physics

Modern physical theories are not arrived at by pure thought (except, maybe, general relativity). The common feature of all physical theories is that they deal with the *information* that we can obtain about physical systems through experiment, or observation. For instance, Maxwell's equations for the electromagnetic field are little more than a succinct summary of the observed properties of electric and magnetic fields and any associated charges and currents. These equations were abstracted from the results of innumerable experiments performed over centuries, along with some clever interpolation on the part of Maxwell. Similar comments could be made about Newton's laws of motion, or thermodynamics. Data is collected, either by casual observation or controlled experiment on, for instance the motion of physical objects, or on the temperature, pressure, volume of solids, liquids, or gases and so on. Within this data, regularities are observed which are best summarized as equations:

$$\mathbf{F} = m\mathbf{a} \qquad \text{— Newton's second law;}$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \qquad \text{— One of Maxwell's equations (Faraday's law);}$$

$$PV = NkT \qquad \text{— Ideal gas law (not really a fundamental law)}$$

What these equations represent are relationships between *information* gained by *observation* of various physical systems and as such are a succinct way of summarizing the relationship between the data, or the information, collected about a physical system. The laws are expressed in a manner consistent with how we understand the world from the view point of classical physics in that the symbols replace precisely known or knowable values of the physical quantities they represent. There is no uncertainty or randomness as a consequence of our ignorance of information about a system implicit in any of these equations. Moreover, classical physics says that this information is a faithful representation of what is 'really' going on in the physical world. These might be called the 'classical laws of information' implicit in classical physics.

What these pre-quantum experimenters were not to know was that the information they were gathering was not refined enough to show that there were fundamental limitations to the accuracy with which they could measure physical properties. Moreover, there was some information that they might have taken for granted as being accessible, simply by trying hard enough, but which we now know could not have been obtained at all! There was in operation unsuspected laws of nature that placed constraints on the information that could be obtained about any physical system. In the absence in the data of any evidence of these laws of nature, the information that was gathered was ultimately organised into mathematical statements that constituted *classical* laws of physics: Maxwell's equations, or Newton's laws of motion. But in the late nineteenth century and on into the twentieth century, experimental evidence began to accrue that suggested that there was something seriously amiss with the classical laws of physics: the data could no longer be fitted to the equations, or, in other words, the theory could not explain the observed experimental results. The choice was clear: either modify the existing theories, or formulate new ones. It was the latter approach that succeeded. Ultimately, what was formulated was a new set of laws of nature, the laws of quantum mechanics, which were essentially a set of laws concerning the *information* that could be gained about the physical world.

These are *not* the same laws as implicit in classical physics. For instance, there are limits on the information that can be gained about a physical system. For instance, if in an experiment we measure the position x of a particle with an accuracy[1] of Δx, and then measure the momentum p of the particle we find that the result for p randomly varies from one run of the experiment to the next, spread over a range Δp. But there is still law here. Quantum mechanics tells us that

$$\Delta x\Delta p \geq \tfrac{1}{2}\hbar \text{ — the Heisenberg Uncertainty Relation}$$

Quantum mechanics also tells us how this information is processed e.g. as a system evolves in time (the Schrödinger equation) or what results might be obtained in in a randomly varying way in a measurement. Quantum mechanics is a theory of information, quantum information theory.

What are the consequences? First, it seems that we lose the apparent certainty and determinism of classical physics, this being replaced by uncertainty and randomness. This randomness is not due to our inadequacies as experimenters — it is built into the very fabric of the physical world. But on the positive side, these quantum laws mean that physical systems can do so much more within these restrictions. A particle with position or momentum uncertain by amounts Δx and Δp means we do not quite know where it is, or how fast it is going, and we can never know this. But

[1] Accuracy indicates closeness to the true value, precision is the repeatability or reproducibility of the measurement.

the particle can be doing a lot more things 'behind the scenes' as compared to a classical particle of precisely defined position and momentum. The result is infinitely richer physics — quantum physics.