MACQUARIE UNIVERSITY

Department of Physics
Division of ICS

PHYSICS 301 Quantum Mechanics (2006)

Assignment 2

Due Date: 29 March 2006

Q1 and Q2 MODIFIED AND/OR CORRECTED IN RESPONSE TO STUDENT COMMENTS

1. (a) Consider a system consisting of two spin half atoms (or qubits), usually referred to as a 'bipartite system'. Each of these atoms can be put into a spin state for which $S_z = \pm\frac{1}{2}\hbar$. Using the notation $0 \leftrightarrow S_z = -\frac{1}{2}\hbar$ and $1 \leftrightarrow S_z = +\frac{1}{2}\hbar$ the four possible states for this system are $|00\rangle, |01\rangle$ and two others.

   (i) What are the other possible states?

   The state of this bipartite system can be manipulated by the use of magnetic fields. Suppose this bipartite system is initially prepared in the state $|\psi\rangle$.

   (ii) What is the symbol in bra-ket notation for the probability amplitude of finding the system in state $\langle mn|$, $m, n = 0$ or 1, given that it was initially prepared in the state $|\psi\rangle$?

   (iii) What is the symbol in bra-ket notation for the probability amplitude of finding the system in the final state $\langle\phi|$ given that it was in one of the intermediate states $|mn\rangle$?

   (iv) What is the symbol in bra-ket notation for the probability amplitude of finding the system in the final state $\langle\phi|$ given that it 'passed through' the intermediate state $|mn\rangle$? (Recall that the system was initially prepared in the state $|\psi\rangle$.)

   (v) Expressed as a sum over appropriate terms, write down an expression for the total probability amplitude of finding the atom in the final state $\langle\phi|$ if the intermediate state through which it passed is *not* observed.

   (b) Suppose the initial and final states are such that the probability amplitudes of part (a)(iv) above have the values $\frac{1}{2}i^{m+n}$, $i = \sqrt{-1}$, if the system has passed through the intermediate state $|mn\rangle, m, n = 0$ or 1.

   (i) What is the total probability of finding the system in the final state $\langle\phi|$ if the intermediate state through which it passed is not observed?

   (ii) Decoherence (as produced by external 'noise' affecting the atoms) is a phyical process which is equivalent to an observer determining which state each atom is in by some suitable measuring process. What is the total probability of finding the system in the final state $\langle\phi|$ if the intermediate state through which it passed is 'observed' through decoherence?

2. A beam of spin half particles (e.g. silver atoms) heading in the y direction passes through a Stern-Gerlach apparatus and the separate beams corresponding to $S_z = \pm\frac{1}{2}\hbar$ then pass through two separate slits. The atoms then strike an observation screen. Using the notation $\pm$ to represent the atom having spin of $S_z = \pm\frac{1}{2}\hbar$, and 1 or 2 to represent the atom positioned at either slit 1 or slit 2, the state in which an atom is positioned at slit 1 and has spin up ($S_z = \frac{1}{2}\hbar$) can be written $|+, 1\rangle$, and similarly for the state in which the atom is positioned at slit 2 with spin down.

(a) Write down in Dirac notation the state vector corresponding to the atom positioned at slit 2 with spin down.

Magnetic fields can be set up at the positions of each slit so as to flip the spin from $+$ to $-$ or vice versa.

(a) Write down, in Dirac notation, the two further states that the atoms be prepared in by use of these fields.

(b) Assign values for the probability amplitudes $\langle +,1|+,1\rangle$, $\langle -,1|-,1\rangle$, $\langle -,1|-,2\rangle$, $\langle +,1|+,2\rangle$ and any four other possible combinations (there are sixteen all together) and justify the values assigned in each case.

(c) The Stern-Gerlach apparatus will prepare an atom in a state $|\psi\rangle$ in which there is equal probability of the atom having spin up and passing through slit 1, or having spin down and passing through slit 2. Which of the four possible states listed below could be used to describe this situation (Assume, for the present, that there are no magnetic fields at the positions of the slits.):

(i) $|\psi\rangle = \dfrac{1}{\sqrt{2}}\big(|-,1\rangle + |+,2\rangle\big).$

(ii) $|\psi\rangle = \dfrac{1}{2}|+,1\rangle - \dfrac{\sqrt{3}}{2}|-,2\rangle\big)$

(iii) $|\psi\rangle = \dfrac{1}{\sqrt{2}}\big(|+,1\rangle - |-,2\rangle\big)$

(iv) $|\psi\rangle = \dfrac{1}{\sqrt{2}}\big(|+,1\rangle + i|-,2\rangle\big)$

In each case, explain why the state could or could not be used to describe the situation.

(d) If we let $|\pm,x\rangle$ represent the state in which the atom is observed to have spin $S_z = \pm\frac{1}{2}\hbar$ and to be at the position x on the observation screen, what do $|\langle +,x|\psi\rangle|^2$ and $|\langle -,x|\psi\rangle|^2$ represent?

(e) The probability of observing the atom to be at position x on the observation screen irrespective of the spin of the atom is the sum of the probability to be observed at position x with spin up plus the probability of it being observed at x but with spin down. Using an appropriate choice of the state $|\psi\rangle$, show that this is given by

$$|\langle +,x|\psi\rangle|^2 + |\langle -,x|\psi\rangle|^2 = \tfrac{1}{2}|\langle +,x|+,1\rangle|^2 + \tfrac{1}{2}|\langle -,x|-,2\rangle|^2.$$

(f) Suppose that a magnetic field at slit 1 flips the spin to spin down. Calculate, once again, $|\langle +,x|\psi\rangle|^2 + |\langle -,x|\psi\rangle|^2$, the detection probability of observing the atom to be at position x on the observation screen irrespective of the spin of the atom. In what way does this result differ from that found in part (e)?

3. (a) The state of a system is given by

$$|\psi\rangle = \tfrac{1}{3}\big[(1-i)|1\rangle + \sqrt{2}e^{-i\pi/4}|2\rangle - (2+i)|3\rangle\big].$$

What is the probability *amplitude* of finding the system, in each of the states $|1\rangle$ and $|2\rangle$? What are the *probabilities* of finding the system in each of these states?

(b) What is the associated bra vector $\langle\psi|$?

(c) Another state of the system is given by

$$|\phi\rangle = \frac{1}{\sqrt{6}}\big[(1-i)|1\rangle - 2i|2\rangle\big].$$

Calculate the inner product $\langle\psi|\phi\rangle$ and determine the probability of observing the system in the state $|\psi\rangle$ given that it was in the state $|\phi\rangle$.

4. An ozone molecule consists of three oxygen atoms arranged to form a triangle. If an electron is attached to the molecule, it can attach itself to any of the oxygen atoms. Let the possible states of the electron be $\{|i\rangle; i = 1, 2, 3\}$ where $|i\rangle$ is the state in which the electron is on the i^{th} atom.

   (a) What is the dimension of the state space for the system?

   (b) The states $\{|i\rangle; i = 1, 2, 3\}$ form a complete, orthonormal basis for the system.

      (i) State mathematically what orthonormality means here.

      (ii) What does it mean that the basis states are complete?

   (c) The system is prepared in the state

$$|\phi\rangle = a(i|1\rangle + (1-i)|2\rangle - |3\rangle).$$

      (i) What value will a have to have in order for this state to be normalized to unity?

      (ii) On which atom is it most likely to find the electron? What is the probability of finding it there?

      (iii) On which atom is it least likely to find the electron?

5. An 'atom' with two energy levels has two states, the excited and ground states, represented by the kets $|e\rangle$ and $|g\rangle$ respectively.

   (a) The states $|e\rangle$ and $|g\rangle$ form a complete orthonormal basis for the system. What does this statement mean?

   (b) The atom is prepared in the state

$$|\psi\rangle = (1 + 2i)|e\rangle + (2 - 4i)|g\rangle.$$

   (i) By calculating $\langle\psi|\psi\rangle$, show that this state is not normalized to unity.

   (ii) Show that the state $|\overline{\psi}\rangle = \frac{1}{5}|\psi\rangle$ is normalized to unity.

   (iii) Using the normalized state, what are the probability amplitudes of observing the atom in the excited state and in the ground state?

   (iv) What are the probabilities of finding the atom in the excited and ground states?