PHYSICS 301 QUANTUM PHYSICS I (2006)

Assignment 1 Solutions

1. (a) Buckyballs are molecules made up of 60 carbon atoms arranged to form a geodesic sphere. Suppose that buckyballs are sent at a velocity of 100 ms^{-1} through a twin slit arrangement in which the slits are separated by a distance of 150 nm. The buckyballs then strike an observation screen placed a further 1.25 m past the slits. Calculate the de Broglie wavelength of the buckyballs (i.e. treat them as if they were quantum objects), and estimate the distance between the maxima of the resultant interference pattern on the screen. Given that a buckyball has a diameter of approximately 1 nm, how does the size of the buckyball compare with the distance between neighbouring maxima of the interference pattern. Is the size of the C_{60} molecule likely to effect the visibility of the interference fringes? At what velocity for the molecules would the interference fringes start to become difficult to detect? [This is an experiment that has been done, though with a diffraction grating rather than just two slits.]

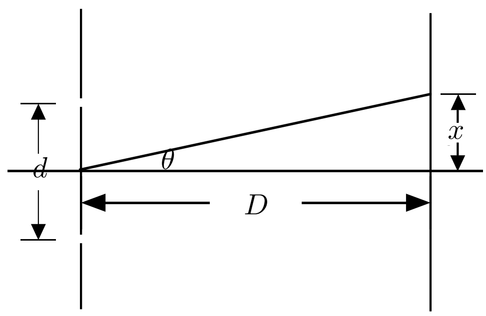
   (b) The buckyballs can be set vibrating by the forces they experience as they pass through the slits. Might it be possible for this to result in the interference pattern disappearing?

SOLUTION

(a) Using the standard result from two-slit interference theory, the phase difference between the two waves reaching the screen at position x is

$$\delta = \frac{2\pi d \sin\theta}{\lambda}.$$

where λ is the wavelength of the waves. The geometry of the situation is illustrated in the accompanying figure.



For the observation screen sufficiently far from the slits, the approximation can be made that

$$\sin\theta \approx \tan\theta = \frac{x}{D}$$

where D is the distance from the screen with the slits to the observation screen. Thus

$$\delta = \frac{2\pi d x}{D\lambda}.$$

Maxima will occur when $\delta = 2n\pi$ where n is an integer. Thus the separation Δx between the maxima will be

$$2\pi\Delta n = \frac{2\pi d\Delta x}{D\lambda}$$

i.e. with $\Delta n = 1$ we get

$$\Delta x = D\lambda/d.$$

The mass of buckball will be $m = 60 \times 12 \times 1.66 \times 10^{-27} = 1.1952 \times 10^{-23}$ kg, so that the wavelength of the de Broglie waves will be

$$\lambda = \frac{h}{p} = \frac{h}{mv} = 5.5458 \times 10^{-12} \text{ m}.$$

Thus the maxima will be separated by a distance Δx given by

$$\Delta x = \frac{D\lambda}{d} = 4.6215 \times 10^{-5} \text{ m}.$$

Thus the separation between fringes will be much larger than the size of each buckball, so the fringes will not be obscured by the size of the molecules.

In order for the fringe separation to be comparable to the molecular dimensions, a fringe separation of 1 nm is required, which is 2.16×10^{-5} smaller than the fringe separation for a velocity of 117 ms^{-1}. As the fringe separation is inversely proportional to the velocity, to produce fringes of 1 nm, it is therefore necessary for the molecules to be moving at a velocity of $100/2.16 \times 10^{-5} = 4.62 \times 10^{6}$ ms^{-1} which is an extremey high velocity that would require the use of a particle accelerator.

(b) The interference pattern will wash out if there is information avaliable on which slit the molecule passes through. If the molecules are set vibrationing in eaxctly the same manner by the two slits, then the fact that they are vibrating would not affect the interference pattern. However, if the molecules are set vibrating in a different manner by each of the slits, then there will be 'which path' information available which would make it possible to determine which slit the molecule passed through. In that case, the interference pattern would be washed out.

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2. In a two slit interference experiment, the following data is collected for the number of detections of particles in the intervals $(n\Delta x, (n+1)\Delta x)$ where $n = -11, -10, \ldots, 9, 10$ and $\Delta x = 1$ mm.

n	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1
detections	4	6	28	22	30	60	169	100	133	148	300

n	0	1	2	3	4	5	6	7	8	9	10
detections	307	147	130	105	161	59	36	21	25	6	3

(a) Construct a histogram that plots $P(x) = \Delta N/N\Delta x$ as a function of x.

(b) Estimate the positions where the probability is a maximum for a particle to strike the screen.

(c) Given that the slit separation is 1 nm, and the observation screen is 1 m from the slits, estimate the wavelength of the waves producing the interference pattern.

(d) If the particles are electrons, what is their velocity?

(e) If the width of each slit is 0.4 nm, draw a rough sketch of what the histogram might look like if the slit through which each electron passed was observed. You will have to take into account both the interference and diffraction patterns patterns produced by the slits.

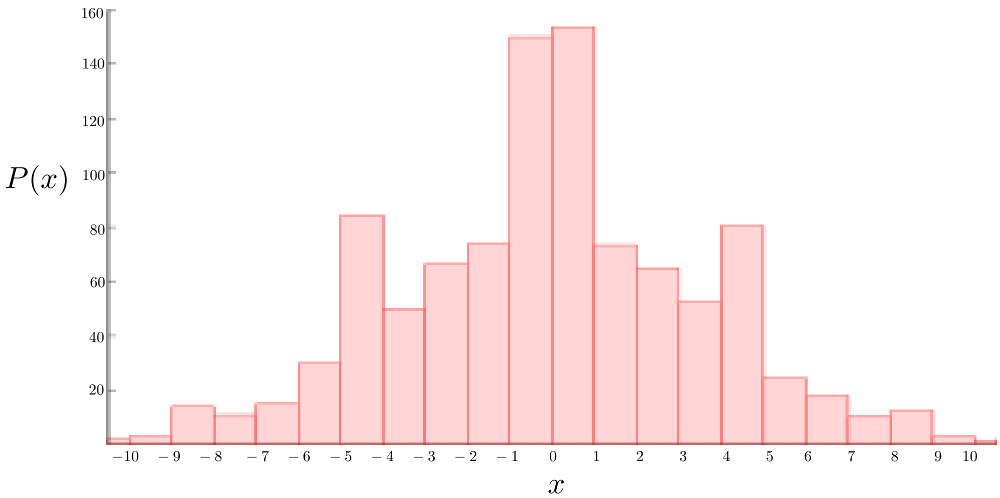
SOLUTION

(a) The total number of detections is $N = 2000$, and with $\Delta x = 10^{-3}$ m, the required values for $P(x) = \Delta N/N\Delta x$ are obtained from the given table simply by dividing the number of detections in each interval by 2. Thus the data to be plotted are as follows

n	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1
$\Delta N/N\Delta x$	2	3	14	11	15	30	84.5	50	66.5	74	150

n	0	1	2	3	4	5	6	7	8	9	10
$\Delta N/N\Delta x$	153.5	73.5	65	52.5	80.5	24.5	18	10.5	12.5	3	1.5

and the required histogram is:

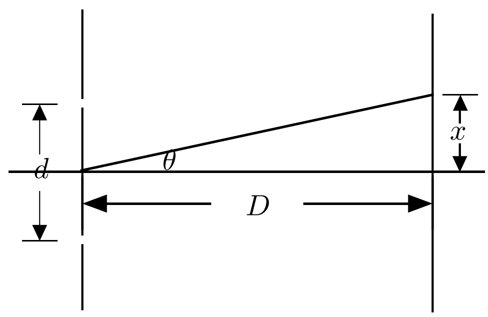


(b) The maxima of the histogram occur at $x = 0$ mm and at $x = \pm 4.5$ mm and again, possibly, at $x = \pm 8.5$ mm.

(c) The phase difference between the waves from the two slits is given by

$$\delta = \frac{2\pi d \sin\theta}{\lambda}.$$

With reference to the diagram of the interference experiment setup, it can be seen that if the observation screen is well removed from the screen with the slits, then $\sin\theta$ can be approximated by



$$\sin\theta \approx \tan\theta = \frac{x}{D}$$

so that

$$\delta \approx \frac{2\pi d x}{D\lambda}.$$

Maxima will occur when $\delta = 2n\pi$ where n is an integer. The central maximum occurs for $n = 0$, while the next nearest maxima occur when $n = \pm 1$. This occurs, from the histogram, at $x = \pm 4.5$ mm. Given that $d = 1$ nm and $D = 1$

m, then with $\delta = 2\pi$ so that $x = 4.5$ mm, then dx/D is 4.5×10^{-12} m^2 and we have

$$\lambda \approx 4.5 \times 10^{-12} \text{ m}.$$

(d) From the de Broglie relation $\lambda = h/p$, and since $p = mv$, then

$$v = h/m\lambda = 1.6 \times 10^8 \text{ ms}^{-1}$$

which is about half the speed of light.

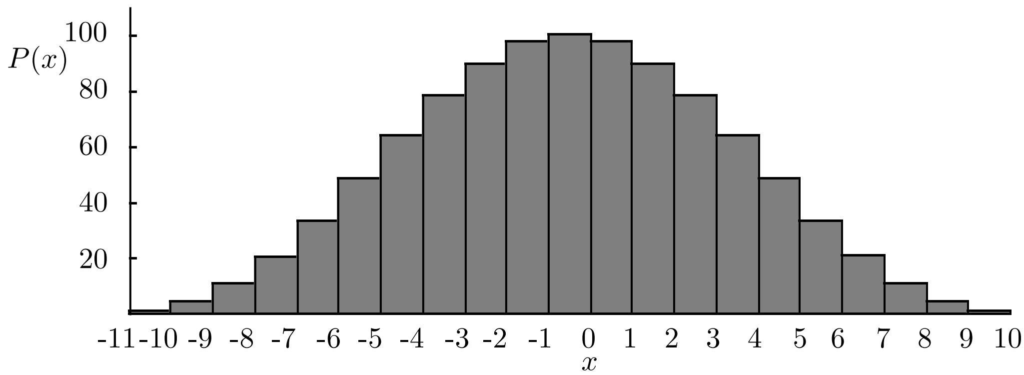
(e) When the slit through which the electrons pass is observed, there is no interference pattern. Instead, the electrons will pass through each slit as if the other were not there. If we are able to determine through which slit each electron passes, *but not through* where *in each slit*, then the electrons will continue to behave like waves when passing through each slit, thereby forming a single slit diffraction pattern whose principal maximum will be directly opposite each slit.

From the theory of diffraction through a pair of slits, the intensity distribution of interference pattern due to two slits of equal width is modulated by the *diffraction* pattern produced by one slit. This diffraction pattern will have a first minimum at $w \sin\theta = \pm\lambda$ where w is the width of the slit. Given that the width of each slit is 0.4 nm, then we would expect the first minimum of the diffraction pattern would occur at

$$x = D\sin\theta = D\lambda/w = 4.5 \times 10^{-12}/0.4 \times 10^{-9} = 0.11 \text{ m}$$

i.e. at a distance of 11 mm fom the central maximum. This is indeed where the diffraction pattern given by the histogram has a minimum.

Thus, if we observe through which slit each electron passes, what will be produced are separate diffraction patterns for each slit which will simply add together. As the slits are separated by a distance of 1 nm, the peaks of these two diffraction patterns will also be separated by 1 nm, and hence on the scale of resolution of the detection of electrons $\delta x = 1$ mm, these two peaks will not be distinguishable. Thus the nett effect will be to fill in the minima of the interference pattern, and diminish the height of the one observable centre peak.



3. In the two slit experiment, it is found that at a point Q directly opposite the midpoint between the two slits, the probability of an electron striking Q if slit 2 is closed is $P_1 = p$.

   (a) What is the probability P_2 of an electron striking Q if slit 1 is closed? [Hint: think symmetry.]

   (b) What would be the probability of an electron striking Q if both slits were open, but the slit through which each electron passed was observed? Explain your reasoning.

   (c) What is the probability *amplitude* of an electron striking the point Q if both slits are open but the slit through which the electrons pass is *not* observed. Hence show that in this case, the probability of an electron striking the point Q is increased as compared to part (b), and determine by what factor this probability is increased.

   (d) At a second point Q' close to Q, it is found that to a good approximation, the values of P_1 and P_2 are the same as their values at Q. However, with both slits open, no electrons are observed to strike Q'. How can this occur?

SOLUTION

(a) By symmetry, the probability $P_2 = p$.

(b) If the slit through which the particle passes is observed, then the probability of the particle arriving at point Q on the screen is just the sum of the two probabilities P_1 and P_2, i.e. $P_1 + P_2 = 2p$.

(c) The probability of the particle arriving at point Q will be just the square of the probability amplitude, i.e. $P_1 = p = |\psi_1|^2$ and $P_2 = p = |\psi_2|^2$. Once again, by symmetry, it must be the case that $\psi_1 = \psi_2 = \sqrt{p}e^{i\phi}$, where ϕ is some phase factor (that we do not know, but do not need to know in this case). If the slit through which the particle passes is not observed, then the probability amplitude of the particle being observed at Q is just the sum of the probability amplitudes associated with the particle passing through one or the other of the two slits, i.e. $\psi_1 + \psi_2 = 2\sqrt{p}e^{i\phi}$. The probability of the particle being observed at Q is then $P_{12} = |\psi_1 + \psi_2|^2 = 4p$, which is twice the probability found when the slit through which the particle passes is observed.

(d) For the point Q' close to Q, the symmetry between ψ_1 and ψ_2 is lost, so that we must put $\psi_1 = \sqrt{p}e^{i\phi_1}$ and $\psi_2 = \sqrt{p}e^{i\phi_2}$ where $\phi_1 \neq \phi_2$ in general. In this case, the probability of a particle being observed at Q' will be

$$|\psi_1 + \psi_2|^2 = p|e^{i\phi_1} + e^{i\phi_2}|^2 = 2p\,[1 + \cos(\phi_2 - \phi_1)]$$

where the cos term represents interference between the contributing probability amplitudes. If $\phi_2 - \phi_1 = \pi$, then this interference term is -1, resulting in a zero probability of detecting a particle. Thus the zero detection probability at Q' is associated with this cancellation due to interference.

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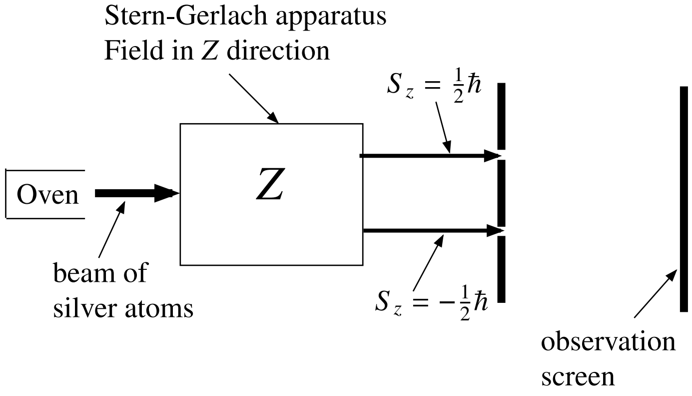
4. In the discussion of the two slit experiment, the fact that the electrons have spin is usually ignored. Suppose however that the experiment is performed in the following fashion:

   A beam of spin half particles (e.g. silver atoms) heading in the y direction passes through a Stern-Gerlach apparatus and the separate beams corresponding to $S_z = \pm\frac{1}{2}\hbar$ then pass through two separate slits.



   The atoms then strike an observation screen. Now consider the following three scenarios, and give an explanation in each case for your answers:

   (a) Will an interference pattern be formed on the screen?
   (b) Suppose a magnetic field is set up across one of the slits of just the right strength to invert the spin of the atoms as they pass through the slit. Will an interference pattern be observed in this case?
   (c) Suppose, instead, that a further Stern-Gerlach apparatus with magnetic field in the x direction is set up in the region after the two slits, but before the observation screen, such that only atoms for which $S_x = \frac{1}{2}\hbar$ direction reach the screen. Will there be any interference pattern observed in this case?

   [Note that the above experiments are more easily done (and have been done) with photons, photon polarization, and polarizers rather than with atoms, spin, and magnetic fields.]

SOLUTION

The important issue in dealing with these questions is whether or not it is possible to determine from the silver atoms that strike the observation screen what the path was that they followed between entering the Stern-Gerlach apparatus and arriving at the screen. If there is no information available to an observer on what the path might be, then it is necessary to suppose that each silver atom 'probes' all the available pathways (in this case two), so that the observed probability of arrival at a particular point on the observation screen will then be the square of the sum of the probability amplitudes for the atoms to follow each pathway, this leading to an interference effect being observed. On the other hand, if there is any such information (even in principle) then there will be no interference pattern.

1. In this case, an atom arriving at the observation screen will carry with it either spin up or spin down, so when it arrives, we will be able to say for certain which slit the atom passed through, so there will be no interference pattern.

2. If we have a magnetic field present at one of the slits, say the slit through which the spin up atoms pass, which rotates this spin to spin down, then when an atom arrives at the observation screen, it will always be observed to have spin down, so we cannot determine through which slit the atom passed. Thus we will expect to see an interference pattern. If, however, the fact that the spin is rotated as it passes through the slit produces a 'back-reaction' on the equipment producing the magnetic field – say a microscopic current pulse occurs through some inductive effect as the spin is rotated, then we would have a record that an atom had passed through this slit, so in this case we would not see an interference pattern. To resolve this issue is a very challenging one.

3. Finally, this magnetic field affects the atoms in the same way irrespective of the slit through which it passes, so even if there is a record that a spin has been flipped, (as discussed in part 2 above) it does so in a situation where the atoms have already passed through the slits so this record cannot provide information on which path. And since, now, the spins of the atoms are the same irrespective of the slit that they passed through, we cannot determine the path that the atoms followed, so the result is an interference pattern will be observed.
