MACQUARIE UNIVERSITY

Department of Physics
Division of ICS

PHYSICS 301 Quantum Mechanics (2006)

Assignment 1

Due Date: 15 March 2006

1. (a) Buckyballs are molecules made up of 60 carbon atoms arranged to form a geodesic sphere. Suppose that buckyballs are sent at a velocity of 100 ms^{-1} through a twin slit arrangement in which the slits are separated by a distance of 150 nm. The buckyballs then strike an observation screen placed a further 1.25 m past the slits. Calculate the de Broglie wavelength of the buckyballs (i.e. treat them as if they were quantum objects), and estimate the distance between the maxima of the resultant interference pattern on the screen. Given that a buckyball has a diameter of approximately 1 nm, how does the size of the buckyball compare with the distance between neighbouring maxima of the interference pattern. Is the size of the C_{60} molecule likely to effect the visibility of the interference fringes? At what velocity for the molecules would the interference fringes start to become difficult to detect? [This is an experiment that has been done, though with a diffraction grating rather than just two slits.]

   (b) The buckyballs can be set vibrating by the forces they experience as they pass through the slits. Might it be possible for this to result in the interference pattern disappearing?

2. In a two slit interference experiment, the following data is collected for the number of detections of particles in the intervals $(n\Delta x, (n{+}1)\Delta x)$ where $n = -11, -10, \ldots, 9, 10$ and $\Delta x = 1$ mm.

n	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1
detections	4	6	28	22	30	60	169	100	133	148	300

n	0	1	2	3	4	5	6	7	8	9	10
detections	307	147	130	105	161	59	36	21	25	6	3

   (a) Construct a histogram that plots $P(x) = \Delta N/N\Delta x$ as a function of x.

   (b) Estimate the positions where the probability is a maximum for a particle to strike the screen.

   (c) Given that the slit separation is 1 nm, and the observation screen is 1 m from the slits, estimate the wavelength of the waves producing the interference pattern.

   (d) If the particles are electrons, what is their velocity?

   (e) If the width of each slit is 0.4 nm, draw a rough sketch of what the histogram might look like if the slit through which each electron passed was observed. You will have to take into account both the interference and diffraction patterns patterns produced by the slits.

3. In the two slit experiment, it is found that at a point Q directly opposite the midpoint between the two slits, the probability of an electron striking Q if slit 2 is closed is $P_1 = p$.

(a) What is the probability P_2 of an electron striking Q if slit 1 is closed?

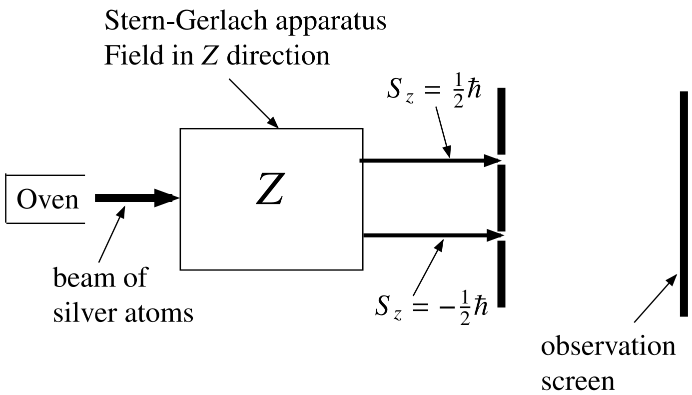
(b) What would be the probability of an electron striking Q if both slits were open, but the slit through which each electron passed was observed? Explain your reasoning.

(c) What is the probability *amplitude* of an electron striking the point Q if both slits are open but the slit through which the electrons pass is *not* observed. Is the probability of an electron striking the point Q increased or decreased as compared to part (b). By what factor does this probability change.

(d) At a second point Q' close to Q, it is found that to a good approximation, the values of P_1 and P_2 are the same as their values at Q. However, with both slits open, no electrons are observed to strike Q'. By using an argument based on probability amplitudes, explain how can this occur.

4. In the discussion of the two slit experiment, the fact that the electrons have spin is usually ignored. Suppose however that the experiment is performed in the following fashion:

A beam of spin half particles (e.g. silver atoms) heading in the y direction passes through a Stern-Gerlach apparatus and the separate beams corresponding to $S_z = \pm\frac{1}{2}\hbar$ then pass through two separate slits.



The atoms then strike an observation screen. Now consider the following three scenarios, and give an explanation in each case for your answers:

(a) Will an interference pattern be formed on the screen?

(b) Suppose a magnetic field is set up across one of the slits of just the right strength to invert the spin of the atoms as they pass through the slit. Will an interference pattern be observed in this case?

(c) Suppose, instead, that a further Stern-Gerlach apparatus with magnetic field in the x direction is set up in the region after the two slits, but before the observation screen, such that only atoms for which $S_x = \frac{1}{2}\hbar$ direction reach the screen. Will there be any interference pattern observed in this case?

[Note that the above experiments are more easily done (and have been done) with photons, photon polarization, and polarizers rather than with atoms, spin, and magnetic fields.]