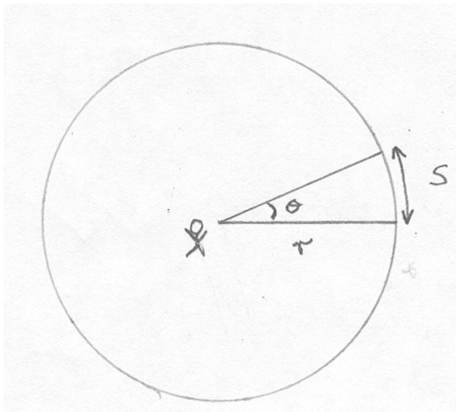
Appendix: The Small Angle Formula

Since they all seem to be equally far away, it looks to us as if they are painted onto a circular dome like in the picture below. You look at two stars and see the angular distance, θ, between the stars. The distance between the stars, "s," depends on both the angle, θ, and the distance to the stars. The bigger the angle, the bigger the size "s." An angle of 90^o corresponds to one quarter of a circle, and an angle of 360^o corresponds to the circumference of the circle. The circumference of a circle is $C=2\pi r$, where r is the radius of the circle. This corresponds to an angle of 360^o. If you increase the angle θ, "s" increases in a similar fashion. This means that the ratio remains the same, or in terms of a formula it means that

$$\frac{s}{\theta} = \text{constant}$$

The largest angle is 360°, but the ratio is still the same, *i.e.*,



$$\frac{C}{360^o} = \text{constant} = \frac{s}{\theta}$$

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We can measure the angle, θ; however, we have to calculate the linear distance "s" by solving the above equation. We get

$$s = \frac{C}{360^o} \cdot \theta$$

Since the circumference C is equal to $2\pi r$, this gives

$$s = \frac{2\pi r}{360^o} \cdot \theta$$

Dividing this by 2π gives

$$\boxed{s = \frac{r}{57.3^o} \cdot \theta}$$

Since the angles in astronomy are generally very small, we often prefer to quote the angles in arc seconds rather than degrees. Since there are 60 arc minutes in one degree, and again 60 arc seconds in one arc minute this gives

$$s = \frac{r \cdot \theta}{57.3 \cdot 60 \cdot 60''}$$

And the final relationship we will use in class then is

$$\boxed{s = \frac{r\theta}{206265''}}$$